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ИНТУИЦИОНИСТСКАЯ ТЕОРИЯ МНОЖЕСТВ

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Книга представляет третью часть 15-летнего исследования автора по интуиционистской теории множеств. В ней приведены исследования автора по интуиционистскому двойному форсингу, созданному в рамках аксиоматической системы **K** интуиционистской теории множеств, предложенной автором в первом томе.

Основная особенность предлагаемого автором интуиционистского двойного форсинга состоит в том, что в нем не используются модели интуиционистской теории множеств **K**.

Книга предназначена для широкого круга математиков, начиная со студентов старших курсов математических специальностей ВУЗов.

Введение

В третьем томе приведены исследования автора по интуиционистскому двойному форсингу, созданному в рамках аксиоматической системы **K** интуиционистской теории множеств, предложенной автором в первом томе ([1] , с. 444), и приведенной далее во втором томе ([2] , с. 4) . Это самый сложный этап 15 − летнего исследования автора по интуиционистской теории множеств.

Основная особенность предлагаемого автором интуиционистского двойного форсинга состоит в том, что в нем не используются модели интуиционистской теории множеств **K**. Существование моделей интуиционистской теории множеств **K** не может быть доказано в аксиоматической системе **K**. Поэтому , мы не можем использовать модели интуиционистской теории множеств **K**, поскольку использование множеств, существование которых не может быть доказано, противоречит принципам интуиционизма. Интуиционисткий метод форсинга основан исключительно на прямом вычислении оценок формул **K** в полных гейтинговых алгебрах, каковыми являются множества открытых множеств частично упорядоченных отношений.

В настоящей книге используется интуиционистское исчисление секвенций $\mathbf{GI}^{+}$ генценовского типа без аксиомы и правила равенства. Большинство доказательств настоящего тома приводятся в виде деревьев формальной системы $\mathbf{GI}^{+}$ генценовского типа без аксиомы и правила равенства, построенных в аксиоматической системе **K**.

В настоящей книге приняты следующие обозначения логических символов: "&" − коньюнкция, " → " − импликация, "¬" − отрицание, " ~ " − эквивалентность.

В третьем томе мы продолжаем использовать обозначения операций и определения предикатов, принятые во втором томе.

Автор выражает особую благодарность академику РАН Юрию Леонидовичу Ершову за 25 – летнюю поддержку своих исследований. Именно благодаря его поддержке стало возможным проведение 15 – летнего исследования автора по интуиционистской теории множеств.

Автор выражает глубокую благодарность бизнесмену Дмитрию Гольдбергу за содействие и помощь в проведении заключительного этапа исследований и подготовке к публикации настоящей книги.

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Глава 1. Генерические множества и ординалы во внутреннем универсуме

§ 1. Плотные множества частично упорядоченных отношений

Определим следующие предикаты:

$^{Df} \vdash \mathrm{Dn}(D,R) \sim \forall r \in |R| ([r]_R \cap D \neq 0)$

(D плотно в R);

$^{Df} \vdash \mathrm{Dn}(D,u,R) \sim \forall r \in u ([r]_R \cap D \neq 0)$

(D u – плотно в R).

Предложение 1 (**K**)

$\mathrm{Tv}(R) \vdash u \cap \overline{_R}\, u = 0$.

Доказательство. Предложение 1 доказано на схеме 1.1.1 (секвенция (1)).

Предложение 2 (**K**)

$\mathrm{Tv}(R) \vdash \mathrm{Dn}(u \cup \overline{_R}\, u, R)$.

Доказательство. Предложение 2 доказано на схеме 1.1.1 (секвенция (2)).

Предложение 3 (**K**)

$u \in \mathrm{O}(R), \mathrm{Tv}(R), \mathrm{Dn}(D \cup \overline{_R}\, u, R) \vdash \mathrm{Dn}(D,u,R)$.

Доказательство. Предложение 3 доказано на схеме 1.1.2 (секвенция (3)).

Предложение 4 (**K**)

$\mathrm{Dn}(D,u,R), \mathrm{Tv}(R) \vdash \mathrm{Dn}(D \cup \overline{_R}\, u, R)$.

Доказательство. Предложение 4 доказано на схемах 1.1.2 и 1.1.3 (секвенция (6) схемы 1.1.3).

§ 2. Генерические множества частично упорядоченных отношений

Определим следующий предикат:

$^{Df}\vdash \mathrm{Gn}(G,R,X) \sim G \subset' |R|\ \&$

$\&\ \forall r \in G\ \forall v\ (\langle v, r \rangle \in R \to \neg\neg\, v \in G)\ \&$

$\&\ \forall u, v \in G\ \neg\neg\exists r \in G\ (\langle u, r \rangle \in R\ \&\ \langle v, r \rangle \in R)\ \&$

$\&\ \forall D \in X\ (\mathrm{Dn}(D,R) \to G \cap D \neq 0)$

(G есть X – генерическое над R множество).

Введем обозначение следующей операции:

$\mathrm{gn}(R) \leftrightharpoons \{\langle r, [r]_R \rangle \mid r \in |R|\}$.

Предложение 5 (**K**)

$\mathrm{Dn}(v,u,R) \vdash u \cap {-}_R\, v = 0$.

Доказательство. Предложение 5 доказано на схеме 1.2.1 (секвенция (1)).

Предложение 6 (**K**)

$\mathrm{Dn}(v,u,R), u \in \mathrm{O}(R) \vdash u \subset' {-}_R {-}_R\, v$.

Доказательство. Предложение 6 доказано на схеме 1.2.1 (секвенция (2)) с использованием предложения 5.

Предложение 7 (**K**)

$\mathrm{Dn}(v,u,R), u \in \mathrm{O}(R), v \subset' |R|, \mathrm{Tv}(R) \vdash u \subset' {-}_R {-}_R \bigcup_{r \in v} [r]_R$.

Доказательство. Предложение 7 доказано на схемах 1.2.1 и 1.2.2 (секвенция (4) схемы 1.2.2) с использованием предложения 6.

В настоящем параграфе принимается следующее обозначение:

$G \leftrightharpoons \mathrm{gn}(R)$.

Предложение 8 (**K**)

$r \in |R| \vdash [\![\check{r} \in G]\!]_R = [r]_R$.

Доказательство. Предложение 8 доказано на схеме 1.2.2 (секвенция (5)).

В настоящем параграфе принимается следующее обозначение :

$G \leftrightharpoons \mathrm{gn}(R)$.

Предложение 9 (**K**)

$D \subset' |R| \vdash [\![G \cap \check{D} \neq 0]\!]_R =_{\overline{R}} {}_{\overline{R}} \bigcup_{r \in D} [\, r \,]_R$.

Доказательство. Предложение 9 доказано на схеме 1.2.2 (секвенция (6)) с использованием предложения 8.

Предложение 10 (**K**)

$\mathrm{Dn}(D,R),\, D \subset' |R|,\, \mathrm{Tv}(R) \vdash |R| \subset' [\![G \cap \check{D} \neq 0]\!]_R$.

Доказательство. Предложение 10 доказано на схеме 1.2.2 (секвенция (7)) с использованием предложений 7 и 9.

Предложение 11 (**K**)

$\mathrm{Tv}(R) \vdash |R| \subset' [\![\forall D \in \check{\mathrm{P}}(|R|)\,(\mathrm{Dn}(D,\check{R}) \to G \cap D \neq 0)]\!]_R$.

Доказательство. Предложение 11 доказано на схеме 1.2.3 (секвенция (8)) с использованием предложения 10.

Доказательство следующего предложения оставляется читателю.

Предложение 12 (**K**)

$\mathrm{Tv}(R) \vdash |R| \subset' [\![\underline{|\check{R}|}_R = |R\check{|}]\!]_R$.

Предложение 13 (**K**)

$\mathrm{Tv}(R) \vdash |R| \subset' [\![\forall t \in G\ \forall v\,(\langle v, t\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R$.

Доказательство. Предложение 13 доказано на схемах 1.2.4 – 1.2.6 (секвенция (13) схемы 1.2.6) с использованием предложений 8 и 12.

Предложение 14 (**K**)

$\mathrm{Tv}(R) \vdash |R| \subset' [\![\forall v \in G\ \forall r \in G\ \neg\neg\exists t \in G(\langle v, t\rangle \in \check{R}\ \&\ \langle r, t\rangle \in \check{R})]\!]_R$.

Доказательство. Предложение 14 доказано на схемах 1.2.6 – 1.2.8 (секвенция (16) схемы 1.2.8) с использованием предложения 8.

Из предложений 11, 13 и 14 следует.

Теорема 1 (**K**)

$\mathrm{Tv}(R) \vdash |R| \subset' [\![\mathrm{Gn}(\mathrm{gn}(R), R, \check{\mathrm{P}}(|R|))]\!]_R$

(теорема о генерическом множестве во внутреннем универсуме).

§ 3. Ординалы во внутреннем универсуме

Введем обозначения следующих операций:

$\underline{\mathcal{R}}_R(u) \leftrightharpoons \underline{\bigcup \{\underline{\mathrm{Sc}}_R(r) \mid r \in \underline{\mathrm{Rg}}_R(u)\}_{R}}_{R}$;

$\underline{\mathrm{Rf}}_R(u) \leftrightharpoons \{\langle h, [\![\mathrm{rf}[h,u]]\!]_R \rangle \mid h \in \mathrm{Dom}(\underline{\mathrm{P}}_R(\underline{\mathrm{V}}_{\underline{\{\{u\}_R\}}_R R}))\}$;

$\hat{\underline{\mathcal{R}}}_R(u) \leftrightharpoons \underline{\bigcup \{\{\langle v, \underline{\mathcal{R}}_R(\underline{f \mid v}_R) \rangle_R \mid f \in \underline{\mathrm{Rf}}_R(v)\}_R \mid v \in \underline{\mathrm{TC}}_R(u)\}_{R}}_{R}$;

$\underline{\mathrm{rg}}_R(u) \leftrightharpoons \underline{\mathcal{R}}_R(\underline{\hat{\mathcal{R}}_R(u) \mid u}_R)$

($\underline{\mathrm{rg}}_R(u)$ есть внутренняя R – операция ранга множества u в $\mathrm{O}(R)$ – универсуме).

Теорема 2 (**K**)

$V^C(u) \vdash |R| \subset' [\![\mathrm{rf}(\hat{\underline{\mathcal{R}}}_R(u), u)]\!]_R$

(теорема об оценке формулы $\mathrm{rf}(\hat{\underline{\mathcal{R}}}_R(u), u)$).

Теорема 3 (**K**)

а) $V^C(r,u) \vdash [\![r \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r))]\!]_R \cap [\![r \in u]\!]_R \subset'_{\overline{R}\,\overline{R}} [\![r \in \underline{\mathrm{rg}}_R(u)]\!]_R$;

б) $V^C(t,u) \vdash [\![t \in \underline{\mathrm{rg}}_R(u)]\!]_R \subset'_{\overline{R}\,\overline{R}} \bigcup\limits_{V^C(r)} ([\![r \in u]\!]_R \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r))]\!]_R)$.

(теорема о внутреннем ранге множества).

Теоремы 2 и 3 доказываются аналогично теоремам 35 и 36 § 6 главы 3 второго тома ([2] , с. 232).

Из утверждения б) теоремы 3 следует

Предложение 15 (**K**)

$V^C(t,u) \vdash [\![t \in \underline{\mathrm{rg}}_R(u)]\!]_R \subset'_{\overline{R}\,\overline{R}} \bigcup\limits_{r \in \mathrm{Dom}(u)} (u(r) \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r))]\!]_R)$.

Следующее предложение доказывается аналогично предложению 83 § 7 главы 3 второго тома ([2] , с. 233).

Предложение 16 (**K**)

$V^C(t,v) \vdash$

$\vdash [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r\check{g}}[v])]\!]_R \subset'_{\overline{R}\,\overline{R}} [\![t \in \underline{\mathrm{Sc}}_R(\mathrm{r\check{g}}[v])]\!]_R$.

Теорема 4 (**K**)

$V^C(Z) \vdash |R| \subset' [\![\underline{\mathrm{rg}}_R(Z) \subset' \mathrm{r\check{g}}[Z]]\!]_R$

(теорема об ограничении внутреннего ранга множества).

Доказательство. В ходе доказательства теоремы 4 используется следующее обозначение :

$Y \rightleftharpoons \{r \in \mathrm{TC}_{\prec}(Z) \cup \{Z\} \mid |R| \subset' [\![\underline{\mathrm{rg}}_R(r) \subset' \mathrm{r\check{g}}[r]]\!]_R\}$.

Следующая секвенция доказана на схемах 1.3.1 – 1.3.3 (секвенция (7) схемы 1.3.3) с использованием предложений 16 и 15, отмеченных на схемах 1.2.2 и 1.2.3 символами [1)] и [2)] соответственно :

$V^C(u), \mathrm{Dom}(u) \subset' Y \vdash |R| \subset' [\![\underline{\mathrm{rg}}_R(u) \subset' \mathrm{r\check{g}}[u]]\!]_R$.

Применение принципа $\prec$– индукции завершает доказательство теоремы 4.

Теорема 5 (**K**)

$V^C(Z) \vdash [\![\mathrm{OR}(Z)]\!]_R \subset' [\![\underline{\mathrm{rg}}_R(Z) = Z]\!]_R$

(теорема о внутреннем ранге ординала).

Доказательство. В ходе доказательства теоремы 5 используется следующее обозначение :

$X \rightleftharpoons \{r \in \mathrm{TC}_{\prec}(Z) \cup \{Z\} \mid [\![\mathrm{OR}(r)]\!]_R \subset' [\![\underline{\mathrm{rg}}_R(r) = r]\!]_R\}$.

Следующая секвенция доказана на схемах 1.3.4 – 1.3.6 (секвенция (12) схемы 1.3.6) с использованием теоремы 3 , следствие утверждения а) которой отмечено на схеме 1.2.4 символом [3)], и предложения 15, отмеченого на схеме 1.2.6 символом [4)] :

$V^C(u), \mathrm{Dom}(u) \subset X \vdash [\![\mathrm{OR}(u)]\!]_R \subset' [\![\underline{\mathrm{rg}}_R(u) = u]\!]_R$.

Применение принципа $\prec$– индукции завершает доказательство теоремы 5.

Теорема 6 (**K**)

$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \subset' \overline{}_R \overline{}_R [\![u \in \mathrm{S\check{c}}(\mathrm{rg}[u])]\!]_R$

(теорема об ограничении оценки формулы $\mathrm{OR}(u)$).

Доказательство. Теорема 6 доказана на схеме 1.3.6 (секвенция

(13)) с использованием теорем 59 и 58, отмеченных на схеме 1.2.6 символами [5)] и [6)] соответственно.

Из теоремы 6 следует

Теорема 7

Если $\varphi(u, v, \ldots, r)$ – отрицательная формула **K**, где $u, v, \ldots, r$ – список всех свободных переменных формулы $\varphi(u, \ldots, v)$, то в **K** доказуема следующая секвенция :

$$V^C(v, \ldots, t) \vdash [\![\neg\neg\exists u(\mathrm{OR}(u) \,\&\, \varphi(u, v, \ldots, t))]\!]_R =$$

$$= -_R -_R \bigcup_{\mathrm{OR}(r)} [\![\varphi(\check{r}, v, \ldots, t)]\!]_R .$$

Схема 1.1.1

$$\begin{array}{c}
r \in |R|,\ \mathrm{Tv}(R) \vdash \neg\neg r \in [r]_R\ ; \\
\neg\neg r \in [r]_R,\ r \in u,\ [r]_R \cap u = 0 \vdash \\
\hline
r \in |R|,\ \mathrm{Tv}(R),\ r \in u,\ [r]_R \cap u = 0 \vdash \\
\hline
\neg\neg r \in |R|\ \&\ [r]_R \cap u = 0,\ \mathrm{Tv}(R),\ r \in u \vdash \\
r \in -_R u \vdash \neg\neg r \in |R|\ \&\ [r]_R \cap u = 0\ ; \\
\hline
r \in -_R u,\ \mathrm{Tv}(R),\ r \in u \vdash \\
\hline
\neg\neg r \in u,\ \neg\neg r \in -_R u,\ \mathrm{Tv}(R) \vdash \\
\hline
\neg\neg r \in u\ \&\ \neg\neg r \in -_R u,\ \mathrm{Tv}(R) \vdash \\
r \in u \cap -_R u \vdash \neg\neg r \in u\ \&\neg\neg r \in -_R u\ ; \\
\hline
r \in u \cap -_R u,\ \mathrm{Tv}(R) \vdash \\
\hline
\mathrm{Tv}(R) \vdash \neg\exists r\,(r \in u \cap -_R u)\ ; \\
\neg\exists r\,(r \in u \cap -_R u) \vdash u \cap -_R u = 0 \\
\hline
\mathrm{Tv}(R) \vdash u \cap -_R u = 0
\end{array} \qquad (1)$$

$$\begin{array}{c}
[r]_R \cap u = 0,\ r \in |R| \vdash \neg\neg r \in -_R u\ ; \\
\neg\neg r \in -_R u,\ r \in [r]_R,\ [r]_R \cap -_R u = 0 \vdash \\
\hline
[r]_R \cap u = 0,\ r \in |R|,\ r \in [r]_R,\ [r]_R \cap -_R u = 0 \vdash \\
\hline
[r]_R \cap u = 0\ \&\ [r]_R \cap -_R u = 0,\ r \in |R|,\ r \in [r]_R \vdash \\
[r]_R \cap (u \cup -_R u) = 0 \vdash [r]_R \cap u = 0\ \&\ [r]_R \cap -_R u = 0\ ; \\
\hline
[r]_R \cap (u \cup -_R u) = 0,\ r \in |R|,\ r \in [r]_R \vdash \\
\hline
[r]_R \cap (u \cup -_R u) = 0,\ r \in |R|,\ \neg\neg r \in [r]_R \vdash \\
r \in |R|,\ \mathrm{Tv}(R) \vdash \neg\neg r \in [r]_R\ ; \\
\hline
[r]_R \cap (u \cup -_R u) = 0,\ r \in |R|,\ \mathrm{Tv}(R) \vdash \\
\hline
r \in |R|,\ \mathrm{Tv}(R) \vdash [r]_R \cap (u \cup -_R u) \neq 0 \\
\hline
\mathrm{Tv}(R) \vdash \\
\vdash \forall r \in |R|\,([r]_R \cap (u \cup -_R u) \neq 0)\ ; \\
\forall r \in |R|\,([r]_R \cap (u \cup -_R u) \neq 0) \vdash \mathrm{Dn}(u \cup -_R u, R) \\
\hline
\mathrm{Tv}(R) \vdash \mathrm{Dn}(u \cup -_R u, R)
\end{array} \qquad (2)$$

Схема 1.1.2

$$
\begin{array}{l}
\mathrm{Tv}(R) \vdash u \cap \underset{R}{-} u = 0\,; \\
\qquad\qquad u \cap \underset{R}{-} u = 0, [r]_R \subset' u \vdash [r]_R \cap \underset{R}{-} u = 0 \\
\hline
\mathrm{Tv}(R), [r]_R \subset' u \vdash [r]_R \cap \underset{R}{-} u = 0 \\
\hline
[r]_R \subset' u, \mathrm{Tv}(R) \vdash [r]_R \cap \underset{R}{-} u = 0
\end{array}
$$

$$
\begin{array}{l}
r \in u, u \in \mathrm{O}(R) \vdash [r]_R \subset' u\,; \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R) \vdash [r]_R \cap \underset{R}{-} u = 0\,; \\
\qquad [r]_R \cap \underset{R}{-} u = 0, [r]_R \cap D = 0 \vdash [r]_R \cap (D \cup \underset{R}{-} u) = 0 \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), [r]_R \cap D = 0 \vdash [r]_R \cap (D \cup \underset{R}{-} u) = 0 \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), [r]_R \cap D = 0, [r]_R \cap (D \cup \underset{R}{-} u) \neq 0 \vdash \\
\qquad r \in |R|, \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash [r]_R \cap (D \cup \underset{R}{-} u) \neq 0\,; \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), [r]_R \cap D = 0, r \in |R|, \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), [r]_R \cap D = 0, \neg\neg\, r \in |R|, \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash \\
\qquad r \in u, u \in \mathrm{O}(R) \vdash \neg\neg\, r \in |R|\,; \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), [r]_R \cap D = 0, \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), \mathrm{Dn}(D \cup \underset{R}{-} u, R), [r]_R \cap D = 0 \vdash \\
\hline
r \in u, u \in \mathrm{O}(R), \mathrm{Tv}(R), \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash [r]_R \cap D \neq 0 \\
\hline
u \in \mathrm{O}(R), \mathrm{Tv}(R), \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash \\
\qquad \vdash \forall r \in u\, ([r]_R \cap D \neq 0)\,; \\
\qquad \forall r \in u\, ([r]_R \cap D \neq 0) \vdash \mathrm{Dn}(D, u, R) \\
\hline
u \in \mathrm{O}(R), \mathrm{Tv}(R), \mathrm{Dn}(D \cup \underset{R}{-} u, R) \vdash \mathrm{Dn}(D, u, R)
\end{array}
\qquad (3)
$$

$$
\begin{array}{l}
q \in [r]_R, \mathrm{Tv}(R) \vdash [q]_R \subset' [r]_R\,; \\
\qquad [q]_R \subset' [r]_R \vdash [q]_R \cap D \subset' [r]_R \cap D \\
\hline
q \in [r]_R, \mathrm{Tv}(R) \vdash [q]_R \cap D \subset' [r]_R \cap D\,; \\
\qquad [q]_R \cap D \subset' [r]_R \cap D, [r]_R \cap D = 0 \vdash [q]_R \cap D = 0 \\
\hline
\qquad q \in [r]_R, \mathrm{Tv}(R), [r]_R \cap D = 0 \vdash [q]_R \cap D = 0 \\
\hline
[q]_R \cap D \neq 0, q \in [r]_R, \mathrm{Tv}(R), [r]_R \cap D = 0 \vdash \\
q \in u, \mathrm{Dn}(D, u, R) \vdash [q]_R \cap D \neq 0\,; \\
\hline
q \in u, \mathrm{Dn}(D, u, R), q \in [r]_R, \mathrm{Tv}(R), [r]_R \cap D = 0 \vdash
\end{array}
\qquad (4)
$$

Схема 1.1.3

$$(4) \Longrightarrow \; q \in u, \operatorname{Dn}(D,u,R), q \in [r]_R, \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash$$

$$\neg\neg\, q \in u, \operatorname{Dn}(D,u,R), q \in [r]_R, \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash$$

$$\neg\neg\, q \in [r]_R \,\&\, \neg\neg\, q \in u, \operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash$$

$$q \in [r]_R \cap u \vdash \neg\neg\, q \in [r]_R \,\&\, \neg\neg\, q \in u\,;$$

$$q \in [r]_R \cap u, \operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash \neg\exists q\,(q \in [r]_R \cap u)\,;$$

$$\neg\exists q\,(q \in [r]_R \cap u) \vdash [r]_R \cap u = 0$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0 \vdash [r]_R \cap u = 0\,;$$

$$[r]_R \cap u = 0, r \in |R| \vdash \neg\neg\, r \in_{\overline{R}} u$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0, r \in |R| \vdash \neg\neg\, r \in_{\overline{R}} u\,;$$

$$r \in |R|, \operatorname{Tv}(R) \vdash \neg\neg\, r \in [r]_R$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0, r \in |R| \vdash \neg\neg\, r \in_{\overline{R}} u \,\&\, \neg\neg\, r \in [r]_R\,;$$

$$r \in_{\overline{R}} u \,\&\, r \in [r]_R, [r]_R \cap_{\overline{R}} u = 0 \vdash$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), [r]_R \cap D = 0, r \in |R|, [r]_R \cap_{\overline{R}} u = 0 \vdash$$

$$[r]_R \cap D = 0 \,\&\, [r]_R \cap_{\overline{R}} u = 0, \operatorname{Dn}(D,u,R), \operatorname{Tv}(R), r \in |R| \vdash \qquad (5)$$

$$[r]_R \cap (D \cup_{\overline{R}} u) = 0 \vdash$$

$$\vdash [r]_R \cap D = 0 \,\&\, [r]_R \cap_{\overline{R}} u = 0\,;$$

$$[r]_R \cap D = 0 \,\&\, [r]_R \cap_{\overline{R}} u = 0, \operatorname{Dn}(D,u,R), \operatorname{Tv}(R), r \in |R| \vdash \; \Longleftarrow (5)$$

$$[r]_R \cap (D \cup_{\overline{R}} u) = 0, \operatorname{Dn}(D,u,R), \operatorname{Tv}(R), r \in |R| \vdash$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R), r \in |R| \vdash$$

$$\vdash [r]_R \cap (D \cup_{\overline{R}} u) \neq 0$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R) \vdash$$

$$\vdash \forall r \in |R|\,([r]_R \cap (D \cup_{\overline{R}} u) \neq 0)\,;$$

$$\forall r \in |R|\,([r]_R \cap (D \cup_{\overline{R}} u) \neq 0) \vdash$$

$$\vdash \operatorname{Dn}(D \cup_{\overline{R}} u, R)$$

$$\operatorname{Dn}(D,u,R), \operatorname{Tv}(R) \vdash \operatorname{Dn}(D \cup_{\overline{R}} u, R) \qquad (6)$$

$$\dfrac{r \in {}_{\overline{R}}v \vdash [\,r\,]_R \cap v = 0}{\neg [\,r\,]_R \cap v = 0,\ r \in {}_{\overline{R}}v \vdash}$$

$$\dfrac{r \in u,\ \mathrm{Dn}(v,u,R) \vdash \neg [\,r\,]_R \cap v = 0\,; \quad \neg [\,r\,]_R \cap v = 0,\ r \in {}_{\overline{R}}v \vdash}{r \in u,\ \mathrm{Dn}(v,u,R),\ r \in {}_{\overline{R}}v \vdash}$$

$$\dfrac{r \in u,\ \mathrm{Dn}(v,u,R),\ r \in {}_{\overline{R}}v \vdash}{\neg\neg\, r \in u,\ \neg\neg\, r \in {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}$$

$$\dfrac{\neg\neg\, r \in u,\ \neg\neg\, r \in {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}{\neg\neg\, r \in u \,\&\, \neg\neg\, r \in {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}$$

$$\dfrac{r \in u \cap {}_{\overline{R}}v \vdash \neg\neg\, r \in u \,\&\, \neg\neg\, r \in {}_{\overline{R}}v\,; \quad \neg\neg\, r \in u \,\&\, \neg\neg\, r \in {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}{r \in u \cap {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}$$

$$\dfrac{r \in u \cap {}_{\overline{R}}v,\ \mathrm{Dn}(v,u,R) \vdash}{\mathrm{Dn}(v,u,R) \vdash \neg\exists r\,(r \in u \cap {}_{\overline{R}}v)\,;}$$

$$\dfrac{\mathrm{Dn}(v,u,R) \vdash \neg\exists r\,(r \in u \cap {}_{\overline{R}}v)\,; \quad \neg\exists r\,(r \in u \cap {}_{\overline{R}}v) \vdash u \cap {}_{\overline{R}}v = 0}{\mathrm{Dn}(v,u,R) \vdash u \cap {}_{\overline{R}}v = 0} \qquad (1)$$

$$(1) \Longrightarrow \quad \mathrm{Dn}(v,u,R) \vdash u \cap {}_{\overline{R}}v \subset' 0\,;$$

$$\dfrac{\mathrm{Dn}(v,u,R) \vdash u \cap {}_{\overline{R}}v \subset' 0\,; \quad u \cap {}_{\overline{R}}v \subset' 0,\ u \in \mathrm{O}(R) \vdash u \subset' ({}_{\overline{R}}v \supset_R 0)}{\mathrm{Dn}(v,u,R),\ u \in \mathrm{O}(R) \vdash u \subset' ({}_{\overline{R}}v \supset_R 0)}$$ [1)]

$$\dfrac{\mathrm{Dn}(v,u,R),\ u \in \mathrm{O}(R) \vdash u \subset' ({}_{\overline{R}}v \supset_R 0)}{\mathrm{Dn}(v,u,R),\ u \in \mathrm{O}(R) \vdash u \subset' {}_{\overline{R}}\,{}_{\overline{R}}v} \qquad (2)$$

$$\dfrac{p \in v,\ v \subset' |\,R\,| \vdash \neg\neg\, p \in |\,R\,|\,; \quad \neg\neg\, p \in |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\neg\, p \in [\,p\,]_R}{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\neg\, p \in [\,p\,]_R\,;}$$

$$\dfrac{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\neg\, p \in [\,p\,]_R\,; \quad p \in v \vdash [\,p\,]_R \subset' \bigcup_{r \in v} [\,r\,]_R}{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\neg\, p \in [\,p\,]_R \,\&\, [\,p\,]_R \subset' \bigcup_{r \in v} [\,r\,]_R\,;}$$

$$\dfrac{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\neg\, p \in [\,p\,]_R \,\&\, [\,p\,]_R \subset' \bigcup_{r \in v} [\,r\,]_R\,; \quad \neg\neg\, p \in [\,p\,]_R \,\&\, [\,p\,]_R \subset' \bigcup_{r \in v} [\,r\,]_R,\ p \notin \bigcup_{r \in v} [\,r\,]_R \vdash}{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R),\ p \notin \bigcup_{r \in v} [\,r\,]_R \vdash}$$

$$\dfrac{p \in v,\ v \subset' |\,R\,|,\ \mathrm{Tv}(R),\ p \notin \bigcup_{r \in v} [\,r\,]_R \vdash}{v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\exists p \in v\,\bigl(\,p \notin \bigcup_{r \in v} [\,r\,]_R\,\bigr)}$$

$$\dfrac{v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash \neg\exists p \in v\,\bigl(\,p \notin \bigcup_{r \in v} [\,r\,]_R\,\bigr)}{v \subset' |\,R\,|,\ \mathrm{Tv}(R) \vdash v \subset' \bigcup_{r \in v} [\,r\,]_R} \qquad (3)$$

$$
\frac{
\begin{array}{c}
(3) \Longrightarrow \quad v \subset' |R|,\, \mathrm{Tv}(R) \vdash v \subset' \bigcup\limits_{r \in v} [r]_R\,; \\
v \subset' \bigcup\limits_{r \in v} [r]_R \vdash \overline{}_R\,\overline{}_R\, v \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in v} [r]_R
\end{array}
}{
v \subset' |R|,\, \mathrm{Tv}(R) \vdash \overline{}_R\,\overline{}_R\, v \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in v} [r]_R
}
$$

$$
\frac{
(2) \Longrightarrow \quad \mathrm{Dn}(v,u,R),\, u \in \mathrm{O}(R) \vdash u \subset' \overline{}_R\,\overline{}_R\, v\,;
}{
\mathrm{Dn}(v,u,R),\, u \in \mathrm{O}(R),\, v \subset' |R|,\, \mathrm{Tv}(R) \vdash u \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in v} [r]_R
} \qquad (4)
$$

$$
\begin{aligned}
\vdash [\![\check{r} \in G]\!]_R &= \bigcup_{y \in \mathrm{Dom}(G)} (G(y) \cap [\![\check{r} = y]\!]_R) = \\
&= \bigcup_{u \in |R|} (G(\check{u}) \cap [\![\check{r} = \check{u}]\!]_R) = \\
&= \bigcup_{u \in |R|} ([u]_R \cap [\![\check{r} = \check{u}]\!]_R)\,;
\end{aligned}
$$

$$
\frac{
r \in |R| \vdash \bigcup\limits_{u \in |R|} ([u]_R \cap [\![\check{r} = \check{u}]\!]_R) = [r]_R
}{
r \in |R| \vdash [\![\check{r} \in G]\!]_R = [r]_R
} \qquad (5)
$$

$$
\begin{aligned}
\vdash [\![G \cap \check{D} \neq 0]\!]_R &= [\![\neg\neg \exists u \in \check{D}\, (u \in G)]\!]_R = \\
&= \overline{}_R\,\overline{}_R \bigcup_{r \in D} [\![\check{r} \in G]\!]_R\,;
\end{aligned}
$$

$$
\frac{
D \subset' |R| \vdash \bigcup\limits_{r \in D} [\![\check{r} \in G]\!]_R = \bigcup\limits_{r \in D} [r]_R \quad \Longleftarrow (5)
}{
D \subset' |R| \vdash [\![G \cap \check{D} \neq 0]\!]_R = \overline{}_R\,\overline{}_R \bigcup\limits_{r \in D} [r]_R
} \qquad (6)
$$

$$
\vdash \neg\neg |R| \in \mathrm{O}(R)\,; \qquad (4) \Rightarrow
$$

$$
\frac{
\Rightarrow \mathrm{Dn}(D, |R|, R),\, \neg\neg |R| \in \mathrm{O}(R),\, D \subset' |R|,\, \mathrm{Tv}(R) \vdash |R| \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in D} [r]_R
}{
\mathrm{Dn}(D, |R|, R),\, D \subset' |R|,\, \mathrm{Tv}(R) \vdash |R| \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in D} [r]_R
}
$$

$$
\frac{
\mathrm{Dn}(D,R) \vdash \mathrm{Dn}(D, |R|, R)\,;
}{
\mathrm{Dn}(D,R),\, D \subset' |R|,\, \mathrm{Tv}(R) \vdash |R| \subset' \overline{}_R\,\overline{}_R \bigcup\limits_{r \in D} [r]_R\,;
}
$$

$$
\frac{
(6)
}{
\mathrm{Dn}(D,R),\, D \subset' |R|,\, \mathrm{Tv}(R) \vdash |R| \subset' [\![G \cap \check{D} \neq 0]\!]_R
} \qquad (7)
$$

С х е м а 1.2.3

$$\mathrm{Tv}(R) \vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' |R|;$$

$$\underline{\mathrm{Dn}(D,R),\ D \subset' |R|,\ \mathrm{Tv}(R) \vdash |R| \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \quad \Longleftarrow (7)}$$

$$\underline{\mathrm{Dn}(D,R),\ D \subset' |R|,\ \mathrm{Tv}(R) \vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R}$$

$$\underline{\mathrm{Dn}(D,R),\ D \subset' |R|,\ \mathrm{Tv}(R), \neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash}$$

$$D \subset' |R|,\ \mathrm{Tv}(R), \neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash$$

$$\vdash \neg \mathrm{Dn}(D,R);$$

$$\underline{\neg \mathrm{Dn}(D,R) \vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R = 0}$$

$$D \subset' |R|,\ \mathrm{Tv}(R), \neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash$$

$$\vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R = 0;$$

$$\llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R = 0,\ 0 \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R,$$

$$\underline{\neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash}$$

$$D \subset' |R|,\ \mathrm{Tv}(R), \neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R,$$

$$0 \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash$$

$$\underline{\vdash 0 \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R;}$$

$$\underline{D \subset' |R|,\ \mathrm{Tv}(R), \neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash}$$

$$D \subset' |R|,\ \mathrm{Tv}(R) \vdash \neg\neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R;$$

$$\neg\neg \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R \vdash$$

$$\underline{\vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R}$$

$$\underline{D \subset' |R|,\ \mathrm{Tv}(R) \vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R}$$

$$\underline{D \in \mathrm{P}(|R|),\ \mathrm{Tv}(R) \vdash \llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \subset' \llbracket G \cap \check{D} \neq 0 \rrbracket_R}$$

$$\underline{D \in \mathrm{P}(|R|),\ \mathrm{Tv}(R) \vdash |R| \subset' (\llbracket \mathrm{Dn}(\check{D}, \check{R}) \rrbracket_R \underset{R}{\supset} \llbracket G \cap \check{D} \neq 0 \rrbracket_R)}$$

$$\underline{D \in \mathrm{P}(|R|),\ \mathrm{Tv}(R) \vdash |R| \subset' \llbracket \mathrm{Dn}(\check{D}, \check{R}) \to G \cap \check{D} \neq 0 \rrbracket_R}$$

$$\mathrm{Tv}(R) \vdash$$

$$\underline{\forall D \in \mathrm{P}(|R|)\,(\,|R| \subset' \llbracket \mathrm{Dn}(\check{D}, \check{R}) \to G \cap \check{D} \neq 0 \rrbracket_R)}$$

$$\mathrm{Tv}(R) \vdash |R| \subset' \bigcap_{D \in \mathrm{P}(|R|)}^{R} \llbracket \mathrm{Dn}(\check{D}, \check{R}) \to G \cap \check{D} \neq 0 \rrbracket_R =$$

$$\underline{= \llbracket \forall D \in \check{\mathrm{P}}(|R|)\,(\mathrm{Dn}(D, \check{R}) \to G \cap D \neq 0) \rrbracket_R}$$

$$\mathrm{Tv}(R) \vdash |R| \subset' \llbracket \forall D \in \check{\mathrm{P}}(|R|)\,(\mathrm{Dn}(D, \check{R}) \to G \cap D \neq 0) \rrbracket_R \qquad (8)$$

$$r \in [v]_R \vdash \neg\neg\langle v, r\rangle \in R\,;$$

$$\neg\neg\langle v, r\rangle \in R,\ \neg\neg\langle r, t\rangle \in R,\ \mathrm{Tv}(R) \vdash \neg\neg\langle v, t\rangle \in R$$

$$r \in [v]_R,\ \neg\neg\langle r, t\rangle \in R,\ \mathrm{Tv}(R) \vdash \neg\neg\langle v, t\rangle \in R$$

$$r \in [v]_R,\ t \in [r]_R,\ \mathrm{Tv}(R) \vdash \neg\neg t \in [v]_R\,;$$

$$\neg\neg t \in [v]_R,\ t \notin [v]_R \vdash$$

$$r \in [v]_R,\ t \in [r]_R,\ \mathrm{Tv}(R),\ t \notin [v]_R \vdash$$

$$r \in [v]_R,\ \mathrm{Tv}(R) \vdash \neg\exists t \in [r]_R\,(t \notin [v]_R)$$

$$r \in [v]_R,\ \mathrm{Tv}(R) \vdash [r]_R \subset' [v]_R$$

$$\vdash [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [r]_R\,;$$

$$r \in [v]_R,\ \mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R$$

$$r \in [v]_R,\ \mathrm{Tv}(R),\ \neg [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R \vdash$$

$$\mathrm{Tv}(R),\ \neg [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R \vdash r \notin [v]_R \qquad (9)$$

$$r \notin [v]_R \vdash \neg\langle v, r\rangle \in R\,;$$

$$\neg\langle v, r\rangle \in R,\ \mathrm{Tv}(R) \vdash |R| \subset' [\![\neg\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R$$

$$r \notin [v]_R,\ \mathrm{Tv}(R) \vdash |R| \subset' [\![\neg\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R\,;$$

$$|R| \subset' [\![\neg\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R,\ \mathrm{Tv}(R) \vdash [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' 0$$

$$r \notin [v]_R,\ \mathrm{Tv}(R) \vdash [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' 0\,;$$

$$[\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' 0 \vdash [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R$$

$$r \notin [v]_R,\ \mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R$$

$$r \notin [v]_R,\ \mathrm{Tv}(R),\ \neg [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R \vdash$$

$$(9)\,;$$

$$\mathrm{Tv}(R),\ \neg [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R \vdash$$

$$\mathrm{Tv}(R) \vdash \neg\neg [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R$$

$$\mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r}\rangle \in \check{R}]\!]_R \subset' [v]_R \qquad (10)$$

$$
\begin{array}{l}
(10) \Longrightarrow \ \mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r} \rangle \in \check{R}]\!]_R \subset' [v]_R \\
\qquad\qquad (5) \underset{v}{\overset{r}{\Longrightarrow}} \ v \in |R| \vdash [\![\check{v} \in G]\!]_R = [v]_R \\
\hline
v \in |R|, \mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r} \rangle \in \check{R}]\!]_R \subset' [\![\check{v} \in G]\!]_R \\
\hline
v \in |R|, \mathrm{Tv}(R) \vdash [r]_R \cap [\![\langle \check{v}, \check{r} \rangle \in \check{R}]\!]_R \subset' \underset{R}{-}\underset{R}{-}[\![\check{v} \in G]\!]_R \\
\hline
v \in |R|, \mathrm{Tv}(R) \vdash [r]_R \subset' ([\![\langle \check{v}, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![\check{v} \in G]\!]_R) \\
\hline
v \in |R|, \mathrm{Tv}(R) \vdash [r]_R \subset' ([\![\langle \check{v}, \check{r} \rangle \in \check{R} \to \neg\neg\, \check{v} \in G]\!]_R \\
\hline
\qquad \mathrm{Tv}(R) \vdash \forall v \in |R| ([r]_R \subset' ([\![\langle \check{v}, \check{r} \rangle \in \check{R} \to \neg\neg\, \check{v} \in G]\!]_R) \\
\hline
\qquad \mathrm{Tv}(R) \vdash [r]_R \subset' \bigcap\nolimits_{v \in |R|}^{R} [\![\langle \check{v}, \check{r} \rangle \in \check{R} \to \neg\neg\, \check{v} \in G]\!]_R \\
\hline
\qquad \mathrm{Tv}(R) \vdash [r]_R \subset' [\![\forall v \in |R\,\check{|}\, (\langle v, \check{r} \rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R = \\
\qquad\qquad = \bigcap\nolimits_{V^C(v)}^{R} [\![v \in |R\,\check{|} \ \&\ \langle v, \check{r} \rangle \in \check{R} \to \neg\neg\, v \in G]\!]_R = \\
\qquad\qquad = \bigcap\nolimits_{V^C(v)}^{R} ([\![v \in |R\,\check{|}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} [\![\neg\neg\, v \in G]\!]_R) = \\
\qquad\qquad = \bigcap\nolimits_{V^C(v)}^{R} ([\![v \in |R\,\check{|}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
\qquad = \bigcap\nolimits_{V^C(v)}^{R} (\underset{R}{-}\underset{R}{-}[\![v \in |R\,\check{|}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} ([\![\underline{|\check{R}|}_{R} = |R\,\check{|}]\!]_R \cap \underset{R}{-}\underset{R}{-}[\![v \in |R\,\check{|}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} ([\![\underline{|\check{R}|}_{R} = |R\,\check{|}]\!]_R \cap \underset{R}{-}\underset{R}{-}[\![v \in \underline{|\check{R}|}_{R}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} (\underset{R}{-}\underset{R}{-}[\![v \in \underline{|\check{R}|}_{R}]\!]_R \cap [\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} ([\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} \underset{R}{-}\underset{R}{-}[\![v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} ([\![\langle v, \check{r} \rangle \in \check{R}]\!]_R \underset{R}{\supset} [\![\neg\neg\, v \in G]\!]_R) = \\
= \bigcap\nolimits_{V^C(v)}^{R} [\![\langle v, \check{r} \rangle \in \check{R} \to \neg\neg\, v \in G]\!]_R = \\
= [\![\forall v\, (\langle v, \check{r} \rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R \\
\hline
\qquad\qquad \mathrm{Tv}(R) \vdash [r]_R \subset' [\![\forall v\, (\langle v, \check{r} \rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R \qquad (11)
\end{array}
$$

$$\begin{array}{l}
(11) \Longrightarrow\ \mathrm{Tv}(R) \vdash [\,r\,]_R \subset' [\![\forall v\,(\langle v, \check{r}\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R \\ \hline
\quad \mathrm{Tv}(R) \vdash |\,R\,| \subset' (\,[\,r\,]_R \underset{R}{\supset} [\![\forall v\,(\langle v, \check{r}\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R\,) \\ \hline
\quad \mathrm{Tv}(R) \vdash \\
\vdash \forall r \in |\,R\,|(\ |\,R\,| \subset' (\,[\,r\,]_R \underset{R}{\supset} [\![\forall v\,(\langle v, \check{r}\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R\,)\,) \\ \hline
\quad \mathrm{Tv}(R) \vdash |\,R\,| \subset' \\
\qquad\qquad \subset' \bigcap\limits_{r \in |R|} (\,[\,r\,]_R \underset{R}{\supset} [\![\forall v\,(\langle v, \check{r}\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R\,) = \\
\qquad\qquad = \bigcap\limits_{r \in |R|} (\,G(\check{r}\,) \underset{R}{\supset} [\![\forall v\,(\langle v, \check{r}\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R\,) = \\
\qquad\qquad = \bigcap\limits_{r \in \mathrm{Dom}(G)} (\,G(\,t\,) \underset{R}{\supset} [\![\forall v\,(\langle v, t\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R\,) = \\
\qquad\qquad = [\![\forall t \in G\ \forall v\,(\langle v, t\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R \\ \hline
\mathrm{Tv}(R) \vdash |\,R\,| \subset' [\![\forall t \in G\ \forall v\,(\langle v, t\,\rangle \in \check{R} \to \neg\neg\, v \in G)]\!]_R
\end{array} \qquad (13)$$

$$\begin{array}{l}
\neg\neg\langle\, u, t\,\rangle \in R, \mathrm{Tv}(R) \vdash |\,R\,| \subset' [\![\neg\neg\langle\, \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\quad \neg\neg\, t \in [\,u\,]_R\,, \mathrm{Tv}(R) \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R\,; \\
\quad \neg\neg\, t \in [\,v\,]_R\,, \mathrm{Tv}(R) \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\quad \neg\neg\, t \in [\,u\,]_R\,, \neg\neg\, t \in [\,v\,]_R\,, \mathrm{Tv}(R) \vdash \\
\qquad\qquad \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R \ \&\ \ |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\neg\neg\, t \in [\,u\,]_R \ \&\ \neg\neg\, t \in [\,v\,]_R\,, \mathrm{Tv}(R) \vdash \\
\qquad\qquad \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R \cap \underset{R}{-}\underset{R}{-}[\![\langle \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\neg\neg\, t \in [\,u\,]_R \cap [\,v\,]_R\,, \mathrm{Tv}(R) \vdash \\
\qquad\qquad \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R \cap \underset{R}{-}\underset{R}{-}[\![\langle \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \subset' \\
\qquad\qquad\qquad \subset' \underset{R}{-}\underset{R}{-}([\![\langle \check{u}, \check{t}\,\rangle \in \check{R}]\!]_R \cap [\![\langle \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R) = \\
\qquad\qquad\qquad = \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R} \ \&\ \langle\, \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\quad t \in [\,u\,]_R \cap [\,v\,]_R\,, \mathrm{Tv}(R) \vdash \\
\qquad\qquad \vdash |\,R\,| \subset' \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R} \ \&\ \langle\, \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R \\ \hline
\quad t \in [\,u\,]_R \cap [\,v\,]_R\,, \mathrm{Tv}(R) \vdash \\
\qquad\qquad \vdash \neg\neg\, t \in \underset{R}{-}\underset{R}{-}[\![\langle \check{u}, \check{t}\,\rangle \in \check{R} \ \&\ \langle\, \check{v}, \check{t}\,\rangle \in \check{R}]\!]_R
\end{array} \qquad (14)$$

Схема 1.2.7

$$\frac{t \in [u]_R \cap [v]_R,\ \mathrm{Tv}(R) \vdash \neg\neg\, t \in [t]_R; \qquad (14)}{t \in [u]_R \cap [v]_R,\ \mathrm{Tv}(R) \vdash \neg\neg\, t \in [t]_R \mathbin{\&} \neg\neg\, t \in \underset{R}{-}\underset{R}{-} \llbracket \langle \check{u}, \check{t} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{t} \rangle \in \check{R} \rrbracket_R}$$

$$\frac{t \in [u]_R \cap [v]_R,\ \mathrm{Tv}(R) \vdash \neg\neg\, t \in [t]_R \cap \underset{R}{-}\underset{R}{-} \llbracket \langle \check{u}, \check{t} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{t} \rangle \in \check{R} \rrbracket_R \subset' \underset{R}{-}\underset{R}{-}([t]_R \cap \llbracket \langle \check{u}, \check{t} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{t} \rangle \in \check{R} \rrbracket_R) \subset' \underset{R}{-}\underset{R}{-} \bigcup_{r \in |R|} ([r]_R \cap \llbracket \langle \check{u}, \check{r} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{r} \rangle \in \check{R} \rrbracket_R)}{t \in [u]_R \cap [v]_R,\ \mathrm{Tv}(R) \vdash \neg\neg\, t \in \underset{R}{-}\underset{R}{-} \bigcup_{r \in |R|} ([r]_R \cap \llbracket \langle \check{u}, \check{r} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{r} \rangle \in \check{R} \rrbracket_R)}$$

$$\frac{}{\mathrm{Tv}(R) \vdash \forall t \in [u]_R \cap [v]_R (\neg\neg\, t \in \underset{R}{-}\underset{R}{-} \bigcup_{r \in |R|} ([r]_R \cap \llbracket \langle \check{u}, \check{r} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{r} \rangle \in \check{R} \rrbracket_R))}$$

$$\frac{\mathrm{Tv}(R) \vdash [u]_R \cap [v]_R \subset' \underset{R}{-}\underset{R}{-} \bigcup_{r \in |R|} ([r]_R \cap \llbracket \langle \check{u}, \check{r} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{r} \rangle \in \check{R} \rrbracket_R) = \underset{R}{-}\underset{R}{-} \bigcup_{r \in |R|} (G(\check{r}) \cap \llbracket \langle \check{u}, \check{r} \rangle \in \check{R} \mathbin{\&} \langle \check{v}, \check{r} \rangle \in \check{R} \rrbracket_R) = \underset{R}{-}\underset{R}{-} \bigcup_{t \in \mathrm{Dom}(G)} (G(t) \cap \llbracket \langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R} \rrbracket_R) = \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R}{\mathrm{Tv}(R) \vdash [u]_R \cap [v]_R \subset' \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R}$$

$$\frac{}{\mathrm{Tv}(R) \vdash [u]_R \subset' ([v]_R \underset{R}{\supset} \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R)}$$

$$\frac{}{\mathrm{Tv}(R) \vdash \forall v \in |R| ([u]_R \subset' ([v]_R \underset{R}{\supset} \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R))}$$

$$\frac{\mathrm{Tv}(R) \vdash [u]_R \subset' \bigcap_{v \in |R|}^{R} ([v]_R \underset{R}{\supset} \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R) = \bigcap_{v \in |R|}^{R} (G(\check{v}) \underset{R}{\supset} \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle \check{v}, t \rangle \in \check{R}) \rrbracket_R) = \bigcap_{r \in \mathrm{Dom}(G)}^{R} (G(r) \underset{R}{\supset} \llbracket \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle r, t \rangle \in \check{R}) \rrbracket_R) = \llbracket \forall r \in G\ \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle r, t \rangle \in \check{R}) \rrbracket_R}{\mathrm{Tv}(R) \vdash [u]_R \subset' \llbracket \forall r \in G\ \neg\neg \exists t \in G(\langle \check{u}, t \rangle \in \check{R} \mathbin{\&} \langle r, t \rangle \in \check{R}) \rrbracket_R \quad (15)}$$

$$\frac{(15) \Rightarrow \mathrm{Tv}(R) \vdash [u]_R \subset' [\![\forall r \in G\, \neg\neg\exists t \in G(\langle \check{u}, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R}{\mathrm{Tv}(R) \vdash\ \vdash |R| \subset' ([u]_R \underset{R}{\supset} [\![\forall r \in G\, \neg\neg\exists t \in G(\langle \check{u}, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R)}$$

$$\frac{}{\mathrm{Tv}(R) \vdash \forall u \in |R| (|R| \subset' ([u]_R \underset{R}{\supset} [\![\forall r \in G\, \neg\neg\exists t \in G(\langle \check{u}, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R))}$$

$$\mathrm{Tv}(R) \vdash$$

$$\vdash |R| \subset' \bigcap_{u \in |R|}^{R} ([u]_R \underset{R}{\supset} [\![\forall r \in G\, \neg\neg\exists t \in G(\langle \check{u}, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R) =$$

$$= \bigcap_{u \in |R|}^{R} (G(\check{u}) \underset{R}{\supset} [\![\forall r \in G\, \neg\neg\exists t \in G(\langle \check{u}, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R) =$$

$$= \bigcap_{v \in \mathrm{Dom}(G)|}^{R} (G(v) \underset{R}{\supset} [\![\forall r \in G\, \neg\neg\exists t \in G(\langle v, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R) =$$

$$= [\![\forall v \in G\, \forall r \in G\, \neg\neg\exists t \in G(\langle v, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R$$

$$\mathrm{Tv}(R) \vdash$$

$$\vdash |R| \subset' [\![\forall v \in G\, \forall r \in G\, \neg\neg\exists t \in G(\langle v, t \rangle \in \check{R}\ \&\ \langle r, t \rangle \in \check{R})]\!]_R \qquad (16)$$

Схема 1.3.1

$$
\begin{array}{l}
V^{C}(v) \vdash \mathrm{tr}(\mathrm{rg}[v]) ; \qquad \mathrm{tr}(\mathrm{rg}[v]) \vdash |R| \subset' [\![\mathrm{tr}(\mathrm{r\check{g}}[v])]\!]_R \\
\hline
V^{C}(v) \vdash |R| \subset' [\![\mathrm{tr}(\mathrm{r\check{g}}[v])]\!]_R \\
\hline
V^{C}(t,v) \vdash [\![t \in \mathrm{r\check{g}}[v]]\!]_R \subset' \\
\qquad \subset' [\![t \in \mathrm{r\check{g}}[v]]\!]_R \cap |R| \subset' [\![t \in \mathrm{r\check{g}}[v]]\!]_R \cap [\![\mathrm{tr}(\mathrm{r\check{g}}[v])]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \\
\hline
V^{C}(t,v) \vdash [\![t \in \mathrm{r\check{g}}[v]]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \\
\hline
V^{C}(t,v) \vdash \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' \neg_R [\![t \in \mathrm{r\check{g}}[v]]\!]_R ; \\
V^{C}(t,v) \vdash \neg_R [\![t \in \mathrm{r\check{g}}[v]]\!]_R \cap [\![t \in \underline{\mathrm{rg}}_R(v)]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' 0 \\
\hline
V^{C}(t,v) \vdash \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![t \in \mathrm{rg}_R(v)]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' 0 \\
\hline
V^{C}(t,v) \vdash \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' \neg_R [\![t \in \underline{\mathrm{rg}}_R(v)]\!]_R \qquad (1)
\end{array}
$$

$$
\begin{array}{ll}
 & V^{C}(t,v) \vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \notin \underline{\mathrm{rg}}_R(v) \to t = \underline{\mathrm{rg}}_R(v)]\!]_R = \\
 & \qquad = [\![t \notin \underline{\mathrm{rg}}_R(v)]\!]_R \underset{R}{\supset} [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R = \\
 & \qquad = \neg_R [\![t \in \underline{\mathrm{rg}}_R(v)]\!]_R \underset{R}{\supset} [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \\
\cline{2-2}
 & V^{C}(t,v) \vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' \neg_R [\![t \in \underline{\mathrm{rg}}_R(v)]\!]_R \underset{R}{\supset} [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \\
\cline{2-2}
(1); & V^{C}(t,v) \vdash \neg_R [\![t \in \underline{\mathrm{rg}}_R(v)]\!]_R \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \\
\hline
 & V^{C}(t,v) \vdash \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \cap \\
 & \qquad \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \quad (2)
\end{array}
$$

$$
\begin{array}{ll}
 & V^{C}(t,v) \vdash [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \\
\cline{2-2}
(2); & V^{C}(t,v) \vdash [\![t = \underline{\mathrm{rg}}_R(v)]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \cap \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \subset' 0 \\
\hline
\multicolumn{2}{l}{V^{C}(t,v) \vdash \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' 0} \\
\hline
\multicolumn{2}{l}{V^{C}(t,v) \vdash [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' \neg_R \neg_R [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R =} \\
\multicolumn{2}{l}{\qquad = [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R} \\
\hline
\multicolumn{2}{l}{V^{C}(t,v) \vdash [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R} \\
\multicolumn{2}{l}{v \in \mathrm{Dom}(u) \subset' Y \vdash |R| \subset' [\![\underline{\mathrm{rg}}_R(v) \subset' \mathrm{r\check{g}}[v]]\!]_R ;} \\
\hline
\multicolumn{2}{l}{v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash |R| \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R} \\
\hline
\multicolumn{2}{l}{v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\check{g}}[v]]\!]_R \qquad (3)}
\end{array}
$$

$$
\begin{array}{l}
(3)\,; \qquad V^{C}(t,v) \vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![\mathrm{OR}(t)]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\breve{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \qquad (4)
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad \dfrac{V^{C}(v) \vdash \mathrm{OR}(\mathrm{rg}[v])\,; \qquad \mathrm{OR}(\mathrm{rg}[v]) \vdash |R| \dot{\subset} [\![\mathrm{OR}(\mathrm{r\breve{g}}[v])]\!]_R}{} \\
(4)\,; \qquad V^{C}(v) \vdash |R| \subset' [\![\mathrm{OR}(\mathrm{r\breve{g}}[v])]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\breve{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \cap |R| \subset' \\
\qquad\qquad \subset' [\![t \subset' \mathrm{r\breve{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r\breve{g}}[v])]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' [\![t \subset' \mathrm{r\breve{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r\breve{g}}[v])]\!]_R\,; \\
\qquad V^{C}(t,v) \vdash [\![t \subset' \mathrm{r\breve{g}}[v]]\!]_R \cap [\![\mathrm{OR}(t)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r\breve{g}}[v])]\!]_R \subset' \\
\qquad\qquad \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v])]\!]_R \quad {}^{1)} \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v])]\!]_R\,; \\
\vdash |R| \subset' [\![\underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v]) = \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v])]\!]_R \cap |R| \subset' \\
\qquad\qquad \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v])]\!]_R \cap [\![\underline{\mathrm{Sc}}_R(\mathrm{r\breve{g}}[v]) = \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \subset' \\
\qquad\qquad \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \subset' Y, V^{C}(t,v) \vdash \\
\vdash [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v))]\!]_R \subset' {}_{\overline{R}}\,{}_{\overline{R}} [\![t \in \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \qquad (5)
\end{array}
$$

$$
\begin{array}{l}
\vdash [\![t \in \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \subset' \bigcup\limits_{r \in \mathrm{Sc}(\mathrm{rg}[v])} [\![t = \check{r}]\!]_R\,; \\
v \in \mathrm{Dom}(u) \vdash \bigcup\limits_{r \in \mathbf{Sc}(\mathbf{rg}[v])} [\![t = \check{r}]\!]_R \subset' \bigcup\limits_{y \in \mathrm{Dom}(u)} \; \bigcup\limits_{r \in \mathrm{Sc}(\mathrm{rg}[y])} [\![t = \check{r}]\!]_R \\
\hline
v \in \mathrm{Dom}(u) \vdash [\![t \in \check{\mathrm{Sc}}(\mathrm{rg}[v])]\!]_R \subset' \bigcup\limits_{y \in \mathrm{Dom}(u)} \; \bigcup\limits_{r \in \mathrm{Sc}(\mathrm{rg}[y])} [\![t = \check{r}]\!]_R \qquad (6)
\end{array}
$$

$$\neg\neg\, r \in \mathrm{rg}[u] \vdash \llbracket t = \check{r} \rrbracket_R \subset' \bigcup_{v \in \mathrm{rg}[u]} \llbracket t = \check{v} \rrbracket_R \,;\; \vdash \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R = \bigcup_{v \in \mathrm{rg}[u]} \llbracket t = \check{v} \rrbracket_R$$

$$\neg\neg\, r \in \mathrm{rg}[u] \vdash \llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$r \in \mathrm{Sc}(\mathrm{rg}[y]),\, y \in \mathrm{Dom}(u) \vdash \neg\neg\, r \in \mathrm{rg}[u]\,;$$

$$r \in \mathrm{Sc}(\mathrm{rg}[y]),\, y \in \mathrm{Dom}(u) \vdash \llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$y \in \mathrm{Dom}(u) \vdash \forall r \in \mathrm{Sc}(\mathrm{rg}[y])\,(\llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$y \in \mathrm{Dom}(u) \vdash \bigcup_{r \in \mathrm{Sc}(\mathrm{rg}[y])} \llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$\vdash \forall y \in \mathrm{Dom}(u)\,(\bigcup_{r \in \mathrm{Sc}(\mathrm{rg}[y])} \llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$(6)\,; \qquad \vdash \bigcup_{y \in \mathrm{Dom}(u)} \bigcup_{r \in \mathrm{Sc}(\mathrm{rg}[y])} \llbracket t = \check{r} \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$v \in \mathrm{Dom}(u) \vdash \llbracket t \in \mathrm{Sc}\check{(}\mathrm{rg}[v]) \rrbracket_R \subset' \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$(5)\,; \quad v \in \mathrm{Dom}(u) \vdash \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{Sc}\check{(}\mathrm{rg}[v]) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$v \in \mathrm{Dom}(u) \subset' Y,\, V^C(t,v) \vdash \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$v \in \mathrm{Dom}(u),\, V^C(u) \vdash V^C(v)\,;$$

$$v \in \mathrm{Dom}(u) \subset' Y,\, V^C(t,u) \vdash \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$\mathrm{Dom}(u) \subset' Y,\, V^C(t,u) \vdash$$

$$\vdash \forall v \in \mathrm{Dom}(u)\,(\llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$\mathrm{Dom}(u) \subset' Y,\, V^C(t,u) \vdash \bigcup_{v \in \mathrm{Dom}(u)} \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$\mathrm{Dom}(u) \subset' Y,\, V^C(t,u) \vdash \frac{}{R}\frac{}{R} \bigcup_{v \in \mathrm{Dom}(u)} \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$V^C(t,u) \vdash \llbracket t \in \underline{\mathrm{rg}}_R(u) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \bigcup_{v \in \mathrm{Dom}(u)} \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(v)) \rrbracket_R \;^{2)}\,;$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' Y \vdash \llbracket t \in \underline{\mathrm{rg}}_R(u) \rrbracket_R \subset' \frac{}{R}\frac{}{R} \llbracket t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' Y \vdash \llbracket t \in \underline{\mathrm{rg}}_R(u) \rrbracket_R \subset' \llbracket \neg\neg\, t \in \mathrm{r\check{g}}[u] \rrbracket_R$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' Y \vdash |R| \subset' (\llbracket t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$V^C(u),\, \mathrm{Dom}(u) \subset' Y \vdash$$

$$\vdash \forall t\,(V^C(u) \to |R| \subset' (\llbracket t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$V^C(u),\, \mathrm{Dom}(u) \subset' Y \vdash |R| \subset' \bigcap_{V^C(u)}^{R} (\llbracket t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in \mathrm{r\check{g}}[u] \rrbracket_R)$$

$$V^C(u),\, \mathrm{Dom}(u) \subset' Y \vdash |R| \subset' \llbracket \underline{\mathrm{rg}}_R(u) \subset' \mathrm{r\check{g}}[u] \rrbracket_R \qquad (7)$$

$$
\begin{array}{l}
V^{C}(r,u) \vdash \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \llbracket \mathrm{OR}(r) \rrbracket_{R}\,; \\
r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket \mathrm{OR}(r) \rrbracket_{R} \subset' \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R} \\
\hline
V^{C}(r,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R}
\end{array}
\qquad (8)
$$

$$
\begin{array}{l}
(8)\,; \qquad V^{C}(r) \vdash \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{Sc}}_{R}(\underline{\mathrm{rg}}_{R}(r)) \rrbracket_{R} \\
\hline
V^{C}(r,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \\
\vdash \llbracket r \dot{\in} u \rrbracket_{R} \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{Sc}}_{R}(\underline{\mathrm{rg}}_{R}(r)) \rrbracket_{R}\,; \\
V^{C}(r,u) \vdash \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{Sc}}_{R}(\underline{\mathrm{rg}}_{R}(r)) \rrbracket_{R} \cap \llbracket r \in u \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \quad ^{3)} \\
\hline
V^{C}(r,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \\
\qquad\qquad r \in \mathrm{Dom}(u),\, V^{C}(u) \vdash u(r) \subset' \llbracket r \in u \rrbracket_{R}\,; \\
\hline
V^{C}(r,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash u(r) \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \\
\qquad\qquad r \in \mathrm{Dom}(u),\, V^{C}(u) \vdash V^{C}(r)\,; \\
\hline
V^{C}(u),\, r \in \mathrm{Dom}(u) \subset' X \vdash u(r) \cap \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \\
\hline
V^{C}(u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' (\, u(r) \underset{R}{\supset} \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R}\,) \\
\hline
\quad V^{C}(u),\, \mathrm{Dom}(u) \subset' X \vdash \\
\qquad\qquad \vdash \forall r \in \mathrm{Dom}(u)\, (\, \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' (\, u(r) \underset{R}{\supset} \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R}\,)) \\
\hline
\quad V^{C}(u),\, \mathrm{Dom}(u) \subset' X \vdash \\
\qquad\qquad\qquad \vdash \llbracket \mathrm{OR}(u) \rrbracket_{R} \subset' \bigcap\limits_{r \in \mathrm{Dom}(u)}^{R} (\, u(r) \underset{R}{\supset} \overline{}_{R}\overline{}_{R}\llbracket r \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R}\,)
\end{array}
\qquad (9)
$$

$$
\begin{array}{l}
V^{C}(r,t) \vdash \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R} \cap \llbracket t \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \subset' \overline{}_{R}\overline{}_{R}\llbracket t \in r \rrbracket_{R}\,; \\
V^{C}(r,t,u) \vdash \overline{}_{R}\overline{}_{R}\llbracket t \in r \rrbracket_{R} \cap \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{tr}(u) \rrbracket_{R} \cap \overline{}_{R}\llbracket t \in u \rrbracket_{R} \subset' 0 \\
\hline
V^{C}(r,t,u) \vdash \\
\vdash \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R} \cap \llbracket t \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R} \cap \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{tr}(u) \rrbracket_{R} \cap \overline{}_{R}\llbracket t \in u \rrbracket_{R} \subset' 0 \\
\hline
V^{C}(r,t,u) \vdash \\
\vdash \llbracket \underline{\mathrm{rg}}_{R}(r) = r \rrbracket_{R} \cap \llbracket r \in u \rrbracket_{R} \cap \llbracket \mathrm{tr}(u) \rrbracket_{R} \cap \overline{}_{R}\llbracket t \in u \rrbracket_{R} \subset' \overline{}_{R}\llbracket t \in \underline{\mathrm{rg}}_{R}(u) \rrbracket_{R}
\end{array}
\qquad (10)
$$

$$V^C(t,r) \vdash \overline{}_R \llbracket t \in \underline{\mathrm{rg}}_R(r) \rrbracket_R \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' \llbracket t = \underline{\mathrm{rg}}_R(r) \rrbracket_R\,;$$

$$V^C(t,r) \vdash \llbracket t = \underline{\mathrm{rg}}_R(r) \rrbracket_R \cap \llbracket \underline{\mathrm{rg}}_R(r) = r \rrbracket_R \subset' \llbracket t = r \rrbracket_R$$

$$V^C(t,r) \vdash \overline{}_R \llbracket t \in \underline{\mathrm{rg}}_R(r) \rrbracket_R \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \cap \llbracket \underline{\mathrm{rg}}_R(r) = r \rrbracket_R \subset' \\ \subset' \llbracket t = r \rrbracket_R\,;$$

$$V^C(t,r,u) \vdash \llbracket t = r \rrbracket_R \cap \llbracket r \in u \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \subset' 0$$

$$V^C(t,r,u) \vdash \overline{}_R \llbracket t \in \underline{\mathrm{rg}}_R(r) \rrbracket_R \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \cap \llbracket \underline{\mathrm{rg}}_R(r) = r \rrbracket_R \cap \\ \cap \llbracket r \in u \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \subset' 0$$

$$(10)\,;$$

$$V^C(r,t,u) \vdash \llbracket \underline{\mathrm{rg}}_R(r) = r \rrbracket_R \cap \llbracket r \in u \rrbracket_R \cap \llbracket \mathrm{tr}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \cap \\ \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' 0$$

$$(8)\,;$$

$$V^C(r,t,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket r \in u \rrbracket_R \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \cap \\ \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' 0$$

$$r \in \mathrm{Dom}(u),\; V^C(u) \vdash V^C(r)\,;$$

$$V^C(t,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash \llbracket r \in u \rrbracket_R \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \cap \\ \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' 0$$

$$r \in \mathrm{Dom}(u),\, V^C(u) \vdash u(r) \subset' \llbracket r \in u \rrbracket_R\,;$$

$$V^C(t,u),\, r \in \mathrm{Dom}(u) \subset' X \vdash u(r) \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \cap \\ \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' 0$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' X \vdash \\ \vdash \forall r \in \mathrm{Dom}(u)\, (\, u(r) \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \cap \\ \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R \subset' 0\,)$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' X \vdash \bigcup_{r \in \mathrm{Dom}(u)} (\, u(r) \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R\,) \cap \\ \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \subset' 0$$

$$V^C(t,u),\, \mathrm{Dom}(u) \subset' X \vdash \overline{}_R\, \overline{}_R \bigcup_{r \in \mathrm{Dom}(u)} (\, u(r) \cap \llbracket t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r)) \rrbracket_R\,) \cap \\ \cap \llbracket \mathrm{OR}(u) \rrbracket_R \cap \overline{}_R \llbracket t \in u \rrbracket_R \subset' 0 \qquad (11)$$

$$V^C(t,u) \vdash [\![t \in \underline{\mathrm{rg}}_R(u)]\!]_R \subset' \neg_R \neg_R \bigcup_{r \in \mathrm{Dom}(u)} (\, u(r) \cap [\![t \in \underline{\mathrm{Sc}}_R(\underline{\mathrm{rg}}_R(r))]\!]_R \,) \;^{4)};$$

$$(11);$$

$$V^C(t,u),\ \mathrm{Dom}(u) \subset' X \vdash [\![t \in \underline{\mathrm{rg}}_R(u)]\!]_R \cap [\![\mathrm{OR}(u)]\!]_R \cap \neg_R [\![t \in u]\!]_R \subset' 0$$

$$V^C(t,u),\ \mathrm{Dom}(u) \subset' X \vdash [\![\mathrm{OR}(u)]\!]_R \subset' (\, [\![t \in \underline{\mathrm{rg}}_R(u)]\!]_R \supset_R \neg_R \neg_R [\![t \in u]\!]_R \,)$$

$$V^C(t,u),\ \mathrm{Dom}(u) \subset' X \vdash [\![\mathrm{OR}(u)]\!]_R \subset' [\![t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in u]\!]_R$$

$$V^C(u),\ \mathrm{Dom}(u) \subset' X \vdash \forall t\, (\, V^C(t) \to$$
$$\to [\![\mathrm{OR}(u)]\!]_R \subset' [\![t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in u]\!]_R \,)$$

$$V^C(u),\ \mathrm{Dom}(u) \subset' X \vdash [\![\mathrm{OR}(u)]\!]_R \subset' \bigcap_{V^C(t)}^{R} [\![t \in \underline{\mathrm{rg}}_R(u) \to \neg\neg\, t \in u]\!]_R =$$
$$= [\![\forall t \in \underline{\mathrm{rg}}_R(u)\ \neg\neg\, t \in u]\!]_R = \bigcap_{r \in \mathrm{Dom}(\underline{\mathrm{rg}}_R(u))}^{R} (\, \underline{\mathrm{rg}}_R(u)(r) \supset_R \neg_R \neg_R [\![r \in u]\!]_R \,)$$

$$V^C(u),\ \mathrm{Dom}(u) \subset' X \vdash$$
$$\vdash [\![\mathrm{OR}(u)]\!]_R \subset' \bigcap_{r \in \mathrm{Dom}(\underline{\mathrm{rg}}_R(u))}^{R} (\, \underline{\mathrm{rg}}_R(u)(r) \supset_R \neg_R \neg_R [\![r \in u]\!]_R \,); \qquad (9)$$

$$V^C(u),\ \mathrm{Dom}(u) \subset' X \vdash$$
$$\vdash [\![\mathrm{OR}(u)]\!]_R \subset' \bigcap_{r \in \mathrm{Dom}(\underline{\mathrm{rg}}_R(u))}^{R} (\, \underline{\mathrm{rg}}_R(u)(r) \supset_R \neg_R \neg_R [\![r \in u]\!]_R \,) \cap$$
$$\cap \bigcap_{r \in \mathrm{Dom}(u)}^{R} (\, u(r) \supset_R \neg_R \neg_R [\![r \in \underline{\mathrm{rg}}_R(u)]\!]_R \,)$$

$$V^C(u),\ \mathrm{Dom}(u) \subset' X \vdash [\![\mathrm{OR}(u)]\!]_R \subset' [\![\underline{\mathrm{rg}}_R(u) = u]\!]_R \qquad (12)$$

$$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \subset' [\![\underline{\mathrm{rg}}_R(u) = u]\!]_R \;^{5)};$$
$$V^C(u) \vdash |\, R \,| \subset' [\![\underline{\mathrm{rg}}_R(u) \subset' \mathrm{r}\breve{\mathrm{g}}[u]]\!]_R \;^{6)}$$

$$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \cap |\, R \,| \subset' [\![\underline{\mathrm{rg}}_R(u) = u]\!]_R \cap [\![\underline{\mathrm{rg}}_R(u) \subset' \mathrm{r}\breve{\mathrm{g}}[u]]\!]_R \subset'$$
$$\subset' [\![u \subset' \mathrm{r}\breve{\mathrm{g}}[u]]\!]_R$$

$$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \subset' [\![u \subset' \mathrm{r}\breve{\mathrm{g}}[u]]\!]_R; \qquad V^C(u) \vdash$$
$$\vdash [\![u \subset' \mathrm{r}\breve{\mathrm{g}}[u]]\!]_R \cap [\![\mathrm{OR}(u)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r}\breve{\mathrm{g}}[u])]\!]_R \subset' \neg_R \neg_R [\![u \in \underline{\mathrm{Sc}}_R(\mathrm{r}\breve{\mathrm{g}}[u])]\!]_R =$$
$$= \neg_R \neg_R [\![u \in \mathrm{S}\check{\mathrm{c}}(\mathrm{rg}[u])]\!]_R$$

$$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \cap [\![\mathrm{OR}(\mathrm{r}\breve{\mathrm{g}}[u])]\!]_R \subset' \neg_R \neg_R [\![u \in \mathrm{S}\check{\mathrm{c}}(\mathrm{rg}[u])]\!]_R$$
$$V^C(u) \vdash |\, R \,| \subset' [\![\mathrm{OR}(\mathrm{r}\breve{\mathrm{g}}[u])]\!]_R;$$

$$V^C(u) \vdash [\![\mathrm{OR}(u)]\!]_R \subset' \neg_R \neg_R [\![u \in \mathrm{S}\check{\mathrm{c}}(\mathrm{rg}[u])]\!]_R \qquad (13)$$

Глава 2 Двойной форсинг

§ 1. Гомеоморфные отношения

Определим следующий предикат :

$^{Df}\vdash \mathrm{hm}(Z,R,Q) \sim Z \subset' |R| \times |Q| \,\&$

$$\& \forall u \in \mathrm{O}(R)\, \neg\neg\, Z\,\text{”}\,u \in \mathrm{O}(Q)\, \&$$

$$\& \forall u,v \in \mathrm{O}(R)\,(u \neq v \to Z\,\text{”}\,u \neq Z\,\text{”}\,v)$$

$$\& \forall u,v \in \mathrm{O}(R)\,(\,Z\,\text{”}(\,u \underset{R}{\supset} v\,) = Z\,\text{”}\,u \underset{Q}{\supset} Z\,\text{”}\,v\,)$$

(Z есть гомеоморфное отношение из R в Q)

Лемма 1 (**K**)

$u \in \mathrm{O}(R), \mathrm{hm}(Z,R,Q) \vdash Z\,\text{”}\, \underset{R}{\neg}\, u = \underset{Q}{\neg}\, Z\,\text{”}\,u$

Доказательство. Лемма 1 следует из определений.

Мы можем определить операцию $Z\,\hat{}\,u$ такую, что в **K** будет доказуема следующая секвенция:

$\vdash \neg\neg\, t \in Z\,\hat{}\,u \sim \neg\neg\exists v,r\,(\langle v, r\,\rangle \in u \,\&\, t = \langle\, Z\,\hat{}\,v, Z\,\text{”}r\,\rangle\,).$

Лемма 2 (**K**)

$V^{\mathrm{O}(R)}(u), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash V^{\mathrm{O}(Q)}(Z\,\hat{}\,u)$

Доказательство. Лемма 2 следует из определений.

Предложение 1 (**K**)

$V^{\mathrm{O}(R)}(X), V^{\mathrm{O}(R)}(U), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$

$\vdash Z\,\text{”}\,[\![X = U]\!]_R = [\![Z\,\hat{}\,X = Z\,\hat{}\,U]\!]_Q\,.$

Доказательство. В ходе доказательства предложения 1 используются следующие обозначения :

$A \leftrightharpoons \mathrm{TC}_{\preceq}(\langle X, U\,\rangle) \cup \{\langle X, U\,\rangle\}\,;$

$\mathfrak{R} \leftrightharpoons A_{\preceq}$

(операции $\mathrm{TC}_{\preceq}(u)$ и $A_{\preceq}$ определены в § 2 главы 3 второго тома);

$Y \leftrightharpoons \{t \in \mathrm{TC}_{\preceq}(\langle X, U\,\rangle) \cup \{\langle X, U\,\rangle\} \mid Z\,\text{”}\,[\![t' = t'']\!]_R = [\![Z\,\hat{}\,t' = Z\,\hat{}\,t'']\!]_Q\}$

(операции u' и u'' определены в § 2 главы 3 второго тома).

Следующая секвенция доказана на схемах 2.1.1 – 2.1.4 (секвенция (1) схемы 2.1.4):

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q), \mathrm{O}(\langle u, v\rangle, \Re) \subset' Y \vdash$$
$$\vdash Z\,\text{''}\,[\![u = v]\!]_R = [\![Z\,\hat{}\,u = Z\,\hat{}\,v]\!]_Q .$$

Использование принципа $\preceq -$ индукции завершает доказательство предложения 1.

Предложение 2 (**K**)

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$$
$$\vdash Z\,\text{''}\,[\![u \notin v]\!]_R = [\![Z\,\hat{}\,u \notin Z\,\hat{}\,v]\!]_Q .$$

Доказательство. Предложение 2 доказано на схеме 2.1.4 (секвенция (2)) с использованием предложения 1. Равенство, отмеченное на схеме 2.1.4 символом [1)], следует из предложения 1.

Предложение 3 (**K**)

Если $\varphi(t,r)$ является формулой **K**, содержащей переменные t и r свободно, и не содержащей переменных Z и v, то в **K** доказуема следующая секвенция:

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q),$$
$$\forall t,r\,(V^{\mathrm{O}(R)}(t)\,\&\,V^{\mathrm{O}(R)}(r) \to Z\,\text{''}\,[\![\varphi(t,r)]\!]_R = [\![\varphi(Z\,\hat{}\,t, Z\,\hat{}\,r)]\!]_Q) \vdash$$
$$\vdash Z\,\text{''}\,[\![\neg\exists t \in u\, \varphi(t,v)]\!]_R = [\![\neg\exists r \in Z\,\hat{}\,u\;\; \varphi(r, Z\,\hat{}\,v)]\!]_Q .$$

Доказательство. Предложение 3 доказано на схемах 2.1.5 и 2.1.6 (секвенция (3) схемы 2.1.6).

Аналогично предложению 3 доказывается

Предложение 4 (**K**)

Если $\varphi(t, \dots, r)$ является формулой **K**, не содержащей переменных Z и v, а $t, \dots, r$ – список всех свободных переменных формулы $\varphi(t, \dots, r)$, то в **K** доказуема следующая секвенция:

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q),$$
$$\forall t,r\,(V^{\mathrm{O}(R)}(t)\,\&\,V^{\mathrm{O}(R)}(r) \to Z\,\text{''}\,[\![\varphi(t, \dots, r)]\!]_R = [\![\varphi(Z\,\hat{}\,t, \dots, Z\,\hat{}\,r)]\!]_Q) \vdash$$
$$\vdash Z\,\text{''}\,[\![\neg\exists t \in u\, \varphi(t, \dots, v)]\!]_R = [\![\neg\exists r \in Z\,\hat{}\,u\;\; \varphi(r, \dots, Z\,\hat{}\,v)]\!]_Q .$$

Теорема 8

Если $\varphi(t, \dots, r)$ является отрицательной Δ_0 – формулой **K**, а $u, \dots, v$ список всех свободных переменных формулы $\varphi(u, \dots, v)$, не содержащей переменной Z, то в **K** доказуема следующая секвенция:

$V^{O(R)}(u, \dots, v), \mathrm{hm}(Z, R, Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$
$\vdash Z\,{''}\,[\![\varphi(u, \dots, v)]\!]_R = [\![\varphi(Z^\frown u, \dots, Z^\frown v)]\!]_Q$.

Доказательство. Теорема 8 доказывается с помощью индукции по длине отрицательной Δ_0 – формулы **K** с использованием предложений 1, 2 и 4.

§ 2. Теорема о проекции канонического генерического множества верхнего универсума

Определим следующий предикат :

$$^{Df}\vdash \mathrm{Rt}(f, Q, R) \sim f : |Q| \twoheadrightarrow |R| \,\&$$
$$\& \forall r \in [p]_Q (\langle p, q\rangle \in f \to \neg\neg\exists y \in [q]_R (\langle r, y\rangle \in f))\,\&$$
$$\& \forall r \in [q]_R (\langle p, q\rangle \in f \to \neg\neg\exists t \in [p]_Q (\langle t, r\rangle \in f))\,\&$$

(f есть ретракция из Q в R)

Предложение 5 (**K**)

$\mathrm{Rt}(f, Q, R) \vdash f^{-1}\,{''}u \underset{Q}{\supset} f^{-1}\,{''}v \subset' f^{-1}\,{''}(u \underset{R}{\supset} v)$.

Доказательство. Предложение 5 доказано на схемах 2.2.1 и 2.2.2 (секвенция (3) схемы 2.2.2).

Предложение 6 (**K**)

$\mathrm{Rt}(f, Q, R) \vdash f^{-1}\,{''}(u \underset{R}{\supset} v) \subset' f^{-1}\,{''}u \underset{Q}{\supset} f^{-1}\,{''}v$.

Доказательство. Предложение 7 доказано на схемах 2.2.3 и 2.2.4 (секвенция (5) схемы 2.2.4).

Из предложений 5 и 6 следует

Предложение 7 (**K**)

$\mathrm{Rt}(f,Q,R) \vdash f^{-1}\text{''}(u \underset{R}{\supset} v) = f^{-1}\text{''}u \underset{Q}{\supset} f^{-1}\text{''}v$.

Предложение 8 (**K**)

а) $u \in \mathrm{Sec}(Q), \mathrm{Rt}(f,Q,R) \vdash \neg\neg\, f\text{''}u \in \mathrm{Sec}(R)$;

б) $u \in \mathrm{Sec}(R), \mathrm{Rt}(f,Q,R) \vdash \neg\neg\, f^{-1}\text{''}u \in \mathrm{Sec}(Q)$.

Доказательство. Утверждение а) предложения 8 доказано на схеме 2.2.4 (секвенция (6)). Утверждение б) доказано на схеме 2.2.5 (секвенция (7)).

Предложение 9 (**K**)

а) $u \in \mathrm{O}(Q), \mathrm{Rt}(f,Q,R), \mathrm{Tv}(R) \vdash \neg\neg\, f\text{''}u \in \mathrm{O}(R)$;

б) $u \in \mathrm{O}(R), \mathrm{Rt}(f,Q,R), \mathrm{Tv}(Q) \vdash \neg\neg\, f^{-1}\text{''}u \in \mathrm{O}(Q)$.

Доказательство. Предложение 9 следует из предложения 8 и доказуемости в **K** секвенции:

$\mathrm{Tv}(R) \vdash \mathrm{O}(R) = \mathrm{Sec}(R)$

(том 2, глава 3, § 1).

Введем обозначение следующей операции :

$$
\begin{aligned}
& R \otimes_{\Delta} S \leftrightharpoons \\
& \leftrightharpoons \{ t \in ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \times \Delta) \times ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \times \Delta) \mid \\
& \qquad \mid \neg\neg\exists r,p \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \; \exists \eta,\sigma \in \Delta \, (r \in [p]_R \,\& \\
& \qquad\qquad \& \, r \in [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R \,\& \\
& \qquad\qquad \& \, t = \langle \langle r, \eta \rangle, \langle p, \sigma \rangle \rangle) \}
\end{aligned}
$$

($R \otimes_{\Delta} S$ есть Δ – произведение R и S).

Предложение 10 (**K**)

$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S) \vdash \mathrm{Tv}(R \otimes_{\Delta} S)$.

Доказательство. Предложение 10 следует из определений.

Предложение 11 (**K**)

$$
\begin{aligned}
& f = \{ t \in ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \times \Delta) \times [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \mid \\
& \qquad \mid \neg\neg\exists p \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \; \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \Big\} ,
\end{aligned}
$$

$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S) \vdash \mathrm{Rt}(f, R \otimes_{\Delta} S, R)$.

Доказательство. Предложение 11 следует из определений.

Предложение 12 (**K**)

$$f = \{ t \in (\llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \times \Delta) \times \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \mid$$
$$\mid \neg\neg \exists p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \},$$
$$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S) \vdash \mathrm{hm}(f^{-1}, R, R \otimes_\Delta S).$$

Доказательство. Предложение 12 следует из предложения 7, утверждения б) предложения 9 и предложения 11.

В настоящей книге принимается следующее обозначение:

$C \rightleftharpoons \mathrm{O}(R)$.

В дальнейшем, в целях упрощения записи из антецедентов секвенций, содержащих операции вида $\llbracket \varphi \rrbracket_R$, $\llbracket \psi \rrbracket_Q$, где φ и ψ – формулы **K**, будем опускать формулу $\mathrm{Tv}(R)$.

Предложение 13 (**K**)

$$f = \{ t \in (\llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \times \Delta) \times \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \mid$$
$$\mid \neg\neg \exists p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \},$$
$$\mathrm{Tv}(R), V^{C}(t, S), Q = R \otimes_\Delta S \vdash$$
$$\vdash \llbracket \mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S) \rrbracket_Q = f^{-1} \text{''} \llbracket \mathrm{Dn}(t, S) \rrbracket_R .$$

Доказательство. Предложение 13 следует из предложения 12 и теоремы 8.

До окончания доказательства теоремы 9 принимаются следующие обозначения:

$G \rightleftharpoons \mathrm{gn}(Q)$;

$$f \rightleftharpoons \{ t \in (\llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \times \Delta) \times \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \mid$$
$$\mid \neg\neg \exists p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \}.$$

Введем обозначение следующей операции:

$$\mathrm{Rf}(t, R, \Delta) \rightleftharpoons \{ v \in (\llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \times \Delta) \mid$$
$$\mid \neg\neg \exists r \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R \exists \eta \in \Delta \, (r \in \llbracket \check{\eta} \in t \rrbracket_R \,\&\, v = \langle r, \eta \rangle) \}.$$

Непосредственно из определений следует

Лемма 1 (**K**)

$$\neg\neg\, t \in f^{-1} \text{''} u, \neg\neg \langle t, v \rangle \in f, \mathrm{Fn}(f) \vdash \neg\neg\, v \in u .$$

Предложение 14 (**K**)

$$p \in [\![\mathrm{Dn}(t,S)]\!]_R \cap [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R\,,\ \sigma \in \Delta, V^C(t,S) \vdash$$
$$\vdash \neg\neg\exists r \in [p]_R\ \exists \eta \in \Delta\,(r \in [\![\check{\eta} \in t]\!]_R \cap [\![\neg\neg\,\check{\eta} \in [\check{\sigma}]_S]\!]_R)\,.$$

Доказательство. Предложение 14 следует из доказуемости в **K** секвенции (1) схемы 2.2.6.

Предложение 15 (**K**)

$$V^C(t,S), Q = R \otimes_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_R\,.$$

Доказательство. Предложение 15 доказано на схемах 2.2.7 – 2.2.9 (секвенция (4) схемы 2.2.9) с использованием предложения 13, предложения 14, отмеченного на схеме 2.2.8 символом [2], и леммы 1, следствие которой отмечено на схеме 2.2.7 символом [1].

Предложение 16 (**K**)

$$V^C(t,S), Q = R \otimes_\Delta S \vdash [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q =$$
$$=_{\overline{Q}\,\overline{Q}} \bigcup_{p \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R} \ \bigcup_{\zeta \in \Delta} \left([\langle p, \zeta \rangle]_Q \cap [\![\check{\zeta} \in f^{-1} {}^\frown t]\!]_Q \right).$$

Доказательство предложения 16 несложно и оставляется читателю.

Предложение 17 (**K**)

$$\mathrm{Dn}(\mathrm{Rf}(t,R,\Delta), [\![\mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S)]\!]_Q, Q), V^C(t,S), Q = R \otimes_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_R\,.$$

Доказательство. Предложение 17 следует из предложения 7 § 2 главы 1 и предложения 10 настоящего параграфа.

Предложение 18 (**K**)

$$V^C(t,S), Q = R \otimes_\Delta S, \Delta \neq 0 \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S)]\!]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q\,.$$

Доказательство. Предложение 18 доказано на схемах 2.2.9 – 2.2.11 (секвенция (7) схемы 2.2.11) с использованием предложений 15, 16 и 17.

Из предложения 18 следует

Теорема 9 (**K**)

$V^{C}(S), Q = R \otimes_{\Delta} S, \Delta \neq 0 \vdash$

$\vdash |Q| \subset' \llbracket \mathrm{Gn}(\underline{\mathrm{Rg}}_{Q}(G), f^{-1} \frown S, f^{-1} \frown \check{\mathrm{P}}(|R|)) \rrbracket_{Q}$

(теорема о проекции канонического генерического множества верхнего универсума).

При выполнении условия

$|R| \subset' \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_{R}$

доказательство теоремы 9 упрощается и становится более прозрачным.

Введем обозначение следующей операции:

$R \otimes'_{\Delta} S \leftrightharpoons$

$$\leftrightharpoons \{t \in (|R| \times \Delta) \times (|R| \times \Delta) \mid \neg\neg \exists r, p \in |R| \, \exists \eta, \sigma \in \Delta \, (r \in [p]_{R} \,\&$$

$$\& \, r \in \llbracket \neg\neg \, \check{\eta} \in [\check{\sigma}]_{S} \rrbracket_{R} \,\&$$

$$\& \, t = \langle \langle r, \eta \rangle, \langle p, \sigma \rangle \rangle)\};$$

Предложение 19 (**K**)

$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S), |R| \subset' \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_{R} \vdash \mathrm{Tv}(R \otimes'_{\Delta} S).$

Предложение 20 (**K**)

$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg \exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle)\},$

$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S), |R| \subset' \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_{R} \vdash \mathrm{Rt}(f, R \otimes'_{\Delta} S, R).$

Предложение 21 (**K**)

$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg \exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle)\},$

$\mathrm{Tv}(R), V^{\mathrm{O}(R)}(S), |R| \subset' \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_{R} \vdash \mathrm{hm}(f^{-1}, R, R \otimes'_{\Delta} S).$

Предложение 22 (**K**)

$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg \exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle)\},$

$\mathrm{Tv}(R), V^{C}(t, S), |R| \subset' \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_{R}, Q = R \otimes'_{\Delta} S \vdash$

$\vdash \llbracket \mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S) \rrbracket_{Q} = f^{-1} \text{''} \llbracket \mathrm{Dn}(t, S) \rrbracket_{R}.$

Предложения 19, 20, 21 и 22 доказываются аналогично предложениям 10, 11, 12 и 13.

До окончания настоящего параграфа принимаются следующие обозначения:

$G \rightleftharpoons \mathrm{gn}(R \otimes'_{\Delta} S)$;

$f \rightleftharpoons \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg\exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle\langle p, \sigma \rangle, p \rangle)\}$.

Предложение 23 (**K**)

$p \in [\![\mathrm{Dn}(t,S)]\!]_R, V^C(t,S), |R| \subset' [\![\, |S| = \check{\Delta}]\!]_R, \sigma \in \Delta, \vdash$

$\vdash \neg\neg\exists r \in [p]_R \, \exists \eta \in \Delta \, (r \in [\![\check{\eta} \in t]\!]_R \cap [\![\neg\neg \, \check{\eta} \in [\check{\sigma}]_S]\!]_R)$.

Доказательство. Предложение 23 следует из доказуемости в **K** секвенции (1) схемы 2.2.6.

Предложение 24 (**K**)

$V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_{\Delta} S \vdash$

$\vdash [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1} \frown t \neq 0]\!]_Q =_{\overline{Q}\,\overline{Q}} \bigcup_{p \in |R|} \bigcup_{\zeta \in \Delta} ([\langle p, \zeta \rangle]_Q \cap [\![\check{\zeta} \in f^{-1} \frown t]\!]_Q)$.

Доказательство предложения 24 несложно и оставляется читателю.

Введем обозначение следующей операции:

$\mathrm{Rf}'(t,R,\Delta) \rightleftharpoons$

$\rightleftharpoons \{v \in (|R| \times \Delta) \mid \neg\neg\exists r \in |R| \, \exists \eta \in \Delta \, (r \in [\![\check{\eta} \in t]\!]_R \,\&\, v = \langle r, \eta \rangle)\}$

Предложение 25 (**K**)

$\mathrm{Dn}(\mathrm{Rf}'(t,R,\Delta), [\![\mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S)]\!]_Q, Q), V^C(t,S),$

$|R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_{\Delta} S \vdash$

$\vdash [\![\mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_R$.

Доказательство. Предложение 25 следует из предложения 7 § 2 главы 1 и предложения 19 настоящего параграфа.

Предложение 26 (**K**)

$V^C(t,S), |R| \subset' [\![\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_{\Delta} S \vdash$

$\vdash [\![\mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_R$.

Предложение 27 (**K**)

$V^C(t,S), |R| \subset' [\![\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_{\Delta} S \vdash$

$\vdash [\![\mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S)]\!]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1} \frown t \neq 0]\!]_Q$.

Предложение 28 (**K**)

$V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_{\Delta} S \vdash$

$\vdash [\![\mathrm{Dn}(f^{-1} \frown t, f^{-1} \frown S)]\!]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1} \frown t \neq 0]\!]_Q$.

Предложения 26, 27 и 28 доказаны на схемах 2.2.12 – 2.2.16 с использованием предложений 22, 23 и 24.

Из предложения 28 следует

Теорема 10 (**K**)

$V^C(S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R\,,\; Q = R \otimes'_{\Delta} S \vdash$

$\vdash |Q| \subset' \left[\!\!\left[\mathrm{Gn}(\,\underline{\mathrm{Rg}}_Q(G), Q, f^{-1} {}^\frown \check{\mathrm{P}}(|R|))\right]\!\!\right]_Q .$

§ 3 Теорема о разбиении открытого в $R \otimes'_{\Delta} S$ множества

Введем обозначения следующих операций:

$S[u] \rightleftharpoons \{ r \in \bigcup\bigcup S \mid \neg\neg \langle u, r \rangle \in S \}$;

$\mathring{S} \rightleftharpoons \{ \langle r, \bigcup\limits_{u \in \mathrm{Dom}(S)} (u \cap \bigcup\limits_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} v(r)) \rangle \mid r \in \bigcup_{\prec} \mathrm{Rg}(S) \}$

Предложение 29 (**K**)

$\vdash [\![t \in \mathring{S}]\!]_R = \bigcup\limits_{u \in \mathrm{Dom}(S)} (u \cap \bigcup\limits_{v \in S[u]} [\![t \in v]\!]_R).$

Доказательство. Предложение 29 доказано на схеме 2.3.1 (секвенция (1)).

Введем обозначение следующей операции:

$Z_{\frown S} \rightleftharpoons \bigcap Z - \bigcup(\mathrm{Dom}(S) - Z)$.

Предложение 30 (**K**)

$\vdash Z_{\frown S} \cap [\![t \in \mathring{S}]\!]_R \subset' \bigcup\limits_{y \in S\,''Z} [\![t \in y]\!]_R$.

Доказательство. Предложение 30 доказано на схемах 2.3.1 и 2.3.2 (секвенция (3) схемы 2.3.2) с использованием предложения 29.

Предложение 31 (**K**)

$Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \bigcup\limits_{v \in S\,''Z} [\![t \in v]\!]_R \subset' [\![t \in \mathring{S}]\!]_R$.

Доказательство. Предложение 31 доказано на схемах 2.3.3 (секвенция (4)) с использованием предложения 29.

Предложение 32 (**K**)

$$Z \subset' \mathrm{O}(R), Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap_{\overline{R}\,\overline{R}} \bigcup_{v \in S''Z} [\![t \in v]\!]_R \subset'_{\overline{R}\,\overline{R}} [\![t \in \mathring{S}]\!]_R .$$

Доказательство. Предложение 32 доказано на схемах 2.3.3 и 2.3.4 (секвенция (6) схемы 2.3.4) с использованием предложения 31.

Введем обозначения следующих операций:

$$\mathrm{Dom}(u,r,R) \rightleftharpoons \{ p \in \mathrm{Dom}(u) \mid \neg\neg\, r \in [p]_R \} ;$$

$$[Z]^R \rightleftharpoons \{ [p]_R \mid p \in Z \} .$$

Лемма 1 (**K**)

$$\mathrm{Dom}(u,r,R) \neq 0 \vdash \neg\neg\, r \in \bigcap\, [\mathrm{Dom}(u,r,R)]^R .$$

Доказательство. Лемма 1 доказана на схеме 2.3.4 (секвенция (7)).

Предложение 33 (**K**)

$$r \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \vdash \neg\neg\, r \in \bigcap\, [\mathrm{Dom}(u,r,R)]^R .$$

Доказательство. Предложение 33 доказано на схеме 2.3.4 (секвенция (8)) с использованием леммы 1.

Введем обозначение следующей операции:

$$\mathrm{Sh}(u,R,S) \rightleftharpoons \{ t \in \{ [p]_R \mid p \in \mathrm{Dom}(u) \} \times \{ \underline{[\check{\sigma}]_S}_R \mid \sigma \in \mathrm{Rg}(u) \} \mid$$
$$\mid \neg\neg \exists p, \sigma\, (\langle p, \sigma \rangle \in u \;\&\; t = \langle [p]_R , \underline{[\check{\sigma}]_S}_R \rangle) \} .$$

Лемма 2 (**K**)

$$\vdash \neg\neg\, t \in \mathrm{Sh}(u,R,S) \sim \neg\neg \exists p, \sigma\, (\langle p, \sigma \rangle \in u \;\&\; t = \langle [p]_R , \underline{[\check{\sigma}]_S}_R \rangle).$$

Доказательство. Лемма 2 следует из определения операции $\mathrm{Sh}(u,R,S)$.

Предложение 34 (**K**)

$$\vdash r \notin \bigcup (\mathrm{Dom}(\mathrm{Sh}(u,R,S)) - [\mathrm{Dom}(u,r,R)]^R) .$$

Доказательство. Предложение 34 доказано на схеме 2.3.5 (секвенция (9)).

Предложение 35 (**K**)

$$r \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \vdash \neg\neg\, r \in [\mathrm{Dom}(u,r,R)]^R{}_{\sim \mathrm{Sh}(u,R,S)} .$$

Доказательство. Предложение 35 доказано на схеме 2.3.5 (секвенция (10)) с использованием предложений 33 и 34.

Предложение 36 (**K**)

$$\langle p, \sigma \rangle \in u, r \in [p]_R, V^C(S),$$
$$r \in [\![\neg\neg \check{\delta} \in [\check{\sigma}]_S]\!]_R \vdash \neg\neg r \in_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Sh}(u,R,S)\text{''}[\mathrm{Dom}(u,r,R)]^R} [\![\check{\delta} \in v]\!]_R .$$

Доказательство. Предложение 36 доказано на схеме 2.3.6 (секвенция (11)).

Лемма 2 (**K**)

$$\langle p, \sigma \rangle \in u, r \in [p]_R \vdash \neg\neg r \in \bigcap [\mathrm{Dom}(u,r,R)]^R.$$

Доказательство. Лемма 2 доказана на схеме 2.3.6 (секвенция (12)) с использованием леммы 1.

В целях упрощения записи, из антецедентов всех секвенций настоящего параграфа до доказательства предложения 42 опускаются следующие формулы:

$$Q = R \otimes'_{\Delta} S\,;$$
$$|R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R .$$

Предложение 37 (**K**)

$$\langle r, \delta \rangle \in [\langle p, \sigma \rangle]_Q, \langle p, \sigma \rangle \in u, V^C(S) \vdash \neg\neg r \in_{\overline{R}\,\overline{R}} [\![\check{\delta} \in \mathrm{Sh}\mathring{(}u,R,S)]\!]_R .$$

Доказательство. Предложение 37 доказано на схеме 2.3.7 (секвенция (13)) с использованием предложений 32, 35 и леммы 2.

Предложение 38 (**K**)

$$u \subset' |R| \times \Delta, V^C(S) \vdash$$
$$\vdash \bigcup_{y \in u} [y]_Q \subset' \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap_{\overline{R}\,\overline{R}} [\![\check{\eta} \in \mathrm{Sh}\mathring{(}u,R,S)]\!]_R \times \Delta).$$

Доказательство. Предложение 38 доказано на схеме 2.3.8 (секвенция (14)) с использованием предложения 37.

Введем обозначение следующей операции:

$$\underset{\smile}{R} \leftrightharpoons \{\langle r, [r]_R \rangle \mid r \in |R|\}.$$

Предложение 39 (**K**)

$$v \in \mathrm{Sh}(u,R,S)\text{''}Z, u \subset' |R| \times \Delta \vdash$$
$$\vdash \neg\neg \exists p \in \underset{\smile}{R}^{-1}\text{''}Z\ \ \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]_S}_R).$$

Доказательство. Предложение 39 доказано на схемах 2.3.9 и 2.3.10 (секвенция (16) схемы 2.3.10).

Предложение 40 (**K**)

$p \in \breve{R}^{-1}{}''Z, r \in \bigcap Z \vdash \neg\neg r \in [p]_R$.

Доказательство. Предложение 40 доказано на схемах 2.3.10 и 2.3.11 (секвенция (18) схемы 2.3.11).

Предложение 41 (**K**)

$$\neg\neg r \in \bigcup_{v \in \mathrm{Sh}(u,R,S)''Z} [\![\check{\eta} \in v]\!]_R , u \subset' |R| \times \Delta, r \in \bigcap Z,$$

$$r \in [\![\neg\neg \check{\delta} \in [\check{\eta}]_S]\!]_R , V^C(S), \delta \in \Delta, t = \langle r, \delta \rangle , t \notin \bigcup_{y \in u} [y]_Q \vdash .$$

Доказательство. Предложение 41 доказано на схемах 2.3.11 – 2.3.13 (секвенция (21) схемы 2.3.13) с использованием предложения 39.

Предложение 42 (**K**)

$$u \subset' |R| \times \Delta, V^C(S) \vdash$$

$$\vdash \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \cap \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u},R,S)]\!]_R \times \Delta) \subset'$$

$$\subset' \bigcup_{y \in u} [y]_Q .$$

Доказательство. Предложение 42 доказано на схемах 2.3.13 – 2.3.15 (секвенция (24) схемы 2.3.15) с использованием предложений 30, 35 и 41.

Из предложений 38 и 42 следует

Теорема 11 (**K**)

$$u \in \mathrm{O}(Q), V^C(S), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \vdash$$

$$\vdash \overline{\overline{u}}_{Q\,Q} = \overline{\overline{\bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \cap}}_{Q\,Q}$$

$$\cap \overline{\overline{\bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u},R,S)]\!]_R \times \Delta)}}_{Q\,Q}$$

(теорема о разбиении открытого в $R \otimes'_\Delta S$ множества).

Теорема 12 (**K**)

$$u \in \mathrm{O}(Q), V^C(S), Q = R \otimes_\Delta S \vdash$$

$$\vdash \overline{\overline{u}}_{Q\,Q} = \overline{\overline{\bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \cap}}_{Q\,Q}$$

$$\cap \overline{\overline{\bigcup_{q \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u},R,S)]\!]_R \times \Delta)}}_{Q\,Q}$$

(теорема о разбиении открытого в $R \otimes_\Delta S$ множества).

Теорема 12 доказывается аналогично теореме 11.

Теорема 13 (**K**)

$$u \in \mathrm{O}(Q), V^{C}(S), G = \mathrm{gn}(Q), Q = R \otimes'_{\Delta} S,$$
$$|R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R},$$
$$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg\exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle\langle p, \sigma \rangle, p \rangle)\} \vdash$$
$$\vdash \overline{}_{Q}\, \overline{}_{Q}\, u = \overline{}_{Q}\, \overline{}_{Q} (\mathrm{Dom}(u) \times \Delta) \cap$$
$$\cap \overline{}_{Q}\, \overline{}_{Q} \left[\!\!\left[\underline{\mathrm{Rg}}_{Q}(G) \cap f^{-1} {}^\frown \mathring{\mathrm{Sh}}(u, R, S) \neq 0 \right]\!\!\right]_{Q}.$$

Доказательство. Теорема 13 следует из теоремы 11 и предложения 24.

Теорема 14 (**K**)

$$u \in \mathrm{O}(Q), V^{C}(S), G = \mathrm{gn}(Q), Q = R \otimes_{\Delta} S,$$
$$f = \{t \in ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R} \times \Delta) \times [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R} \mid$$
$$\mid \neg\neg\exists p \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R} \, \exists \sigma \in \Delta \, (t = \langle\langle p, \sigma \rangle, p \rangle)\Big\} \vdash$$
$$\vdash \overline{}_{Q}\, \overline{}_{Q}\, u = \overline{}_{Q}\, \overline{}_{Q} (\mathrm{Dom}(u) \times \Delta) \cap$$
$$\cap \overline{}_{Q}\, \overline{}_{Q} \left[\!\!\left[\underline{\mathrm{Rg}}_{Q}(G) \cap f^{-1} {}^\frown \mathring{\mathrm{Sh}}(u, R, S) \neq 0 \right]\!\!\right]_{Q}.$$

Доказательство. Теорема 14 следует из теоремы 12 и предложения 16.

Предложение 43 (**K**)

$$\mathfrak{A} \in \mathrm{O}(Q), V^{C}(u), V^{C}(v), Q = R \otimes'_{\Delta} S, |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R},$$
$$[\![f^{-1} {}^\frown u = f^{-1} {}^\frown v]\!]_{Q} \cap \mathfrak{A} \subset' \mathfrak{S},$$
$$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg\exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle\langle p, \sigma \rangle, p \rangle)\} \vdash$$
$$\vdash [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A}, R, S)]\!]_{R} \cap [\![u = v]\!]_{R} \subset' [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_{R}.$$

Доказательство. Предложение 43 доказано на схемах 2.3.16 – 2.3.20 (секвенция (29) схемы 2.3.20).

В целях упрощения записи, из антецедентов всех секвенций схем 2.3.16 – 2.3.20 опущены следующие формулы:

$$Q = R \otimes'_{\Delta} S, |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_{R},$$
$$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg\exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle\langle p, \sigma \rangle, p \rangle)\}.$$

Из предложения 43 следует

Предложение 44 (**K**)

$\mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R,$
$[\![f^{-1} {}^\frown u = f^{-1} {}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S},$
$f = \{t \in (|R| \times \Delta) \times |R| \mid \neg\neg\exists p \in |R| \exists \sigma \in \Delta\, (t = \langle\langle p, \sigma \rangle, p \rangle)\} \vdash$
$\vdash [\![u = v]\!]_R \subset' [\![\mathrm{Sh}\mathring{(}\mathfrak{A}, R, S) \subset' \mathrm{Sh}\mathring{(}\mathfrak{S}, R, S)]\!]_R .$

Аналогично предложению 43 доказывается

Предложение 45 (**K**)

$\mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), Q = R \otimes_\Delta S,$
$[\![f^{-1} {}^\frown u = f^{-1} {}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S},$
$f = \{t \in ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \times \Delta) \times [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \mid$
$\mid \neg\neg\exists p \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \exists \sigma \in \Delta\, (t = \langle\langle p, \sigma \rangle, p \rangle)\} \vdash$
$\vdash [\![t \in \mathrm{Sh}\mathring{(}\mathfrak{A}, R, S)]\!]_R \cap [\![u = v]\!]_R \subset' [\![t \in \mathrm{Sh}\mathring{(}\mathfrak{S}, R, S)]\!]_R .$

Из предложения 45 следует

Предложение 46 (**K**)

$\mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), Q = R \otimes_\Delta S,$
$[\![f^{-1} {}^\frown u = f^{-1} {}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S},$
$f = \{t \in ([\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \times \Delta) \times [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \mid$
$\mid \neg\neg\exists p \in [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R \exists \sigma \in \Delta\, (t = \langle\langle p, \sigma \rangle, p \rangle)\} \vdash$
$\vdash [\![u = v]\!]_R \subset' [\![\mathrm{Sh}\mathring{(}\mathfrak{A}, R, S) \subset' \mathrm{Sh}\mathring{(}\mathfrak{S}, R, S)]\!]_R .$

§ 4 Теоремы о каноническом имени элемента верхнего универсума

Аналогично определению операции $\mathrm{rg}[u]$, можно определить операцию $\mathrm{In}(u, H)$, такую, что в **K** доказуема следующая секвенция:

$\vdash \neg\neg\, t \in \mathrm{In}(u, H) \sim \neg\neg\exists v \in \mathrm{Dom}(u)(u(v) \cap H \neq 0 \,\&\, t = \mathrm{In}(v, H)).$

Операцию $\mathrm{In}(u, H)$ будем называть $H-$ интерпретацией u.

Аналогично определению операции $\underline{\mathrm{rg}}_R(u)$, можно определить операцию $\underline{\mathrm{In}}_R(u,H)$, такую, что в $\mathbf{K}$ доказуема следующая секвенция:

$$V^C(t,u,H)\vdash_{\overline{R}\,\overline{R}} [\![t\in \underline{\mathrm{In}}_R(u,H)]\!]_R =$$
$$=_{\overline{R}\,\overline{R}} \bigcup_{V^C(v)} ([\![v\in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![\underline{u(v)}_R \cap H\neq 0]\!]_R \cap$$
$$\cap [\![t=\underline{\mathrm{In}}_R(v,H)]\!]_R).$$

Аналогично определению операции $\underline{\mathrm{V}}_{u_R}$, можно определить операцию $\underline{\mathrm{V}}^C_{u\,R}$, такую, что в $\mathbf{K}$ доказуема следующая секвенция:

$$V^C(t,u)\vdash_{\overline{R}\,\overline{R}} [\![t\in \underline{\mathrm{V}}^C_{u\,R}]\!]_R =_{\overline{R}\,\overline{R}} \bigcup_{V^C(v)} ([\![v\in u]\!]_R \cap [\![t\in \underline{\mathrm{PF}}_R(\underline{\mathrm{V}}^C_{v\,R} \underline{\times}_R C)]\!]_R).$$

Аналогично определению операции $\underline{\mathrm{rg}}_R(u)$ можно определить операцию $\hat{u}^{R,S,Q}$, такую, что в $\mathbf{K}$ доказуема следующая секвенция:

$$\hat{u}^{R,S,Q} =$$
$$=\{\langle r, \bigcup_{v\in\mathrm{Dom}(u)} (\mathrm{Dom}([\![v\in u]\!]_Q)\cap \Big[\!\Big[r=\langle \hat{v}^{R,S,Q}, \mathring{\mathrm{Sh}}([\![v\in u]\!]_Q, R, S)\rangle \Big]\!\Big]_R)\rangle \mid r\in$$
$$\in \mathrm{Dom}(\underline{\mathrm{V}}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{rg}[\check{u}]\,R} \underline{\times}_R \underline{\mathrm{O}}_R(S))\}.$$

Детали определения операций $\mathrm{In}(u,H)$, $\underline{\mathrm{In}}_R(u,H)$, $\underline{\mathrm{V}}^C_{u\,R}$ и $\hat{u}^{R,S,Q}$ оставляются читателю.

Для упрощения записи, буквы R, S и Q в операции $\hat{u}^{R,S,Q}$ будем опускать.

В настоящем параграфе принимаются следующие обозначения:

$Q \rightleftharpoons R\otimes'_\Delta S$;

$f \rightleftharpoons \{t\in(|R|\times\Delta)\times|R| \mid \neg\neg\exists p\in|R|\,\exists\sigma\in\Delta\,(t=\langle\langle p,\sigma\rangle, p\rangle)\}$;

$D \rightleftharpoons \mathrm{O}(R\otimes'_\Delta S)$;

$G \rightleftharpoons \mathrm{gn}(R\otimes'_\Delta S)$.

Если $u\in\mathrm{O}(R)$, то примается следующее обозначение:

$\mathrm{h}u \rightleftharpoons f^{-1}{}''u$.

Если $V^{\mathrm{O}(R)}(u)$, то примается следующее обозначение:

$\mathrm{h}u \rightleftharpoons f^{-1}{}^\frown u$.

Также, для упрощения записи, из антецедентов всех секвенций настоящего параграфа опущена формула:

$|R|\subset' [\![\mathrm{Tv}(S)\,\&\,|S|=\check{\Delta}]\!]_R$.

Из теоремы 13 следует

Предложение 47 (**K**)

$$u \in \mathrm{O}(Q), V^{C}(S) \vdash_{\overline{Q}\,\overline{Q}} u =_{\overline{Q}\,\overline{Q}} f^{-1}\text{"}\,\mathrm{Dom}(u) \cap$$
$$\cap_{\overline{Q}\,\overline{Q}} \left[\!\left[\underline{\mathrm{Rg}}_{Q}(G) \cap f^{-1} {}^{\frown} \mathrm{Sh}\mathring{(}u, R, S) \neq 0 \right]\!\right]_{Q}.$$

Из предложения 44 следует

Предложение 48 (**K**)

$$V^{C}(\hat{t}, \hat{v}), V^{D}(u), V^{C}(S) \vdash [\![\hat{t} = \hat{v}]\!]_{R} \subset'$$
$$\subset' \left[\!\left[\mathrm{Sh}\mathring{(}\,[\![\underline{\mathrm{In}}_{Q}(\mathrm{h}\hat{t}, \underline{\mathrm{Rg}}_{Q}(G)) \in u]\!]_{Q}, R, S) = \mathrm{Sh}\mathring{(}\,[\![\underline{\mathrm{In}}_{Q}(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_{Q}(G)) \in u]\!]_{Q}, R, S) \right]\!\right]_{R}.$$

Предложение 49 (**K**)

$$V^{C}(t), V^{D}(u), V^{C}(S) \vdash [\![t \in \hat{u}]\!]_{R} =$$
$$= \bigcup_{v \in \mathrm{Dom}(u)} (\, \mathrm{Dom}([\![v \in u]\!]_{Q}) \cap \left[\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_{Q}, R, S)\,\rangle \right]\!\right]_{R} \cap$$
$$\cap \bigcup_{r \in \mathrm{Dom}(\hat{u})} [\![t = r]\!]_{R}\,).$$

Доказательство. Предложение 49 доказано на схеме 2.4.1 (секвенция (1)).

Предложение 50 (**K**)

$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}{}_{R}]\!]_{R}),$$
$$v \in \mathrm{Dom}(u), V^{C}(S) \vdash |R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[}u]}}{}_{R}]\!]_{R}.$$

Доказательство. Предложение 50 доказано на схеме 2.4.1 (секвенция (2)).

Предложение 51 (**K**)

$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}{}_{R}]\!]_{R}), V^{C}(t, y, S), V^{D}(u) \vdash$$
$$\vdash \forall v \in \mathrm{Dom}(u)(\,[\![t = \hat{v}]\!]_{R} \cap [\![y \in \underline{\mathrm{O}}_{R}(S)]\!]_{R} \subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y \rangle}_{R} = r]\!]_{R}).$$

Доказательство. Предложение 51 доказано на схемах 2.4.1 и 2.4.2 (секвенция (4) схемы 2.4.2).

Предложение 52 (**K**)

$$V^{C}(t, y, S), V^{D}(u) \vdash_{\overline{R}\,\overline{R}} [\![\underline{\langle t, y \rangle}_{R} \in \hat{u}]\!]_{R} =$$
$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(u)} (\, \mathrm{Dom}([\![v \in u]\!]_{Q}) \cap [\![t = \hat{v}]\!]_{R} \cap [\![y \in \underline{\mathrm{O}}_{R}(S)]\!]_{R} \cap$$
$$\cap \left[\!\left[y = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_{Q}, R, S) \right]\!\right]_{R} \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y \rangle}_{R} = r]\!]_{R}\,).$$

Доказательство. Предложение 52 доказано на схемах 2.4.2 – 2.4.3 (секвенция (6) схемы 2.4.3) с использованием предложения 49.

Предложение 53 (**K**)

$$V^C(t,y,S), V^D(u),$$

$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\breve{(}\mathrm{rg}[r])}}{}_R]\!]_R) \vdash$$

$$\vdash_{\overline{R}\,\overline{R}} [\![\underline{\langle t, y \rangle}_R \in \hat{u}]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![t = \hat{v}]\!]_R \cap$$

$$\cap \left[\!\!\left[y = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R) .$$

Доказательство. Предложение 53 доказано на схеме 2.4.4 (секвенция (7)) с использованием предложений 51 и 52.

Предложение 54 (**K**)

$$|R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S), V^C(\hat{t}), t \in \mathrm{Dom}(u), V^D(u), V^C(\hat{u}) \vdash$$

$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\breve{(}\mathrm{rg}[r])}}{}_R]\!]_R) \vdash$$

$$\vdash \mathrm{Dom}([\![t \in u]\!]_Q) \subset' \left[\!\!\left[\hat{u}(\hat{t}) = \mathrm{Sh}\mathring{(}\,[\![t \in u]\!]_Q, R, S) \rangle \right]\!\!\right]_R .$$

Доказательство. Предложение 54 доказано на схемах 2.4.4 и 2.4.5 (секвенция (9) схемы 2.4.5) с использованием предложения 53.

Предложение 55 (**K**)

$$V^C(S), V^D(t,u),$$

$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\breve{(}\mathrm{rg}[r])}}{}_R]\!]_R),$$

$$\forall v \in \mathrm{Dom}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_R) \vdash$$

$$\vdash_{\overline{Q}\,\overline{Q}} [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = v]\!]_Q) .$$

Доказательство. Предложение 55 доказано на схемах 2.4.6 – 2.4.8 (секвенция (10) схемы 2.4.8) с использованием предложения 53.

Предложение 56 (**K**)

$V^C(S), V^D(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R ,$

$\forall r \in \mathrm{Dom}(u) \, (\, |R| \subset' \underset{R}{\overline{\ }}\,\underset{R}{\overline{\ }}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])_R}}]\!]_R),$

$\forall v \in \mathrm{Dom}(u) \, (\, |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash$

$\vdash \forall t \, (\, V^D(t) \to \underset{Q}{\overline{\ }}\,\underset{Q}{\overline{\ }}[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q = \underset{Q}{\overline{\ }}\,\underset{Q}{\overline{\ }}[\![t \in u]\!]_Q \,) .$

Доказательство. Предложение 56 доказано на схемах 2.4.8 – 2.4.10 (секвенция (12) схемы 2.4.10) с использованием предложений 54, 55 и предложения 47, отмеченного на схеме 2.4.10 символом [1].

Предложение 57 (**K**)

$\forall r \in \breve{\mathrm{Dom}}(Z) \, (\, |R| \subset' \underset{R}{\overline{\ }}\,\underset{R}{\overline{\ }}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])_R}}]\!]_R), V^C(S), V^D(Z) \vdash$

$\vdash \forall r \in \breve{\mathrm{Dom}}(Z) \, (\, |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q) .$

Доказательство. В ходе доказательства предложения 57 используется следующее обозначение :

$Y \leftrightharpoons \{u \in \breve{\mathrm{Dom}}(Z) \,|\, |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G)) = u]\!]_Q\} .$

Предложение 57 доказано на схемах 2.4.11 и 2.4.12 (секвенция (15) схемы 2.4.12) с использованием предложения 56 и принципа $\prec -$ индукции.

Теорема 15 (**K**)

$V^D(U), V^C(S) \vdash |R| \subset' \underset{R}{\overline{\ }}\,\underset{R}{\overline{\ }}[\![\hat{U} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[U])_R}}]\!]_R$

(первая теорема о каноническом имени элемента верхнего универсума).

Доказательство. В ходе доказательства теоремы 15 используется следующее обозначение :

$X \leftrightharpoons \{ \, u \in \mathrm{TC}_\prec(\{U\}) \cup \{U\} \,|\, |R| \subset' \underset{R}{\overline{\ }}\,\underset{R}{\overline{\ }}[\![\hat{u} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[u])_R}}]\!]_R \, \} .$

Следующая секвенция

$V^D(U), V^C(S), V^D(G) \vdash \mathrm{TC}_\prec(\{U\}) \cup \{U\} \subset' X$

доказана на схемах 3.4.12 – 3.4.17 (секвенция (25) схемы 2.4.17) с использованием предложений 49, 50, 52, 57, принципа $\check{\prec} -$ индукции и предложения 48, отмеченного на схеме 2.4.13 символом [2].

Теорема 15 следует из доказуемости в **K** следующих секвенций:

$V^D(U), V^C(S), V^D(G) \vdash \mathrm{TC}_{\prec}(\{U\}) \cup \{U\} \subset' X$;

$\vdash \neg\neg U \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\}$.

Следствие теоремы 15 (**K**)

$V^D(U), V^C(S) \vdash |Q| \subset'_{\overline{Q}\ \overline{Q}} [\![\mathrm{h}\hat{U} \in \mathrm{h}\underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[U])}}_R]\!]_Q$.

Теорема 16 (**K**)

$V^D(U) \vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{U}, \underline{\mathrm{Rg}}_Q(G)) = U]\!]_Q$

(вторая теорема о каноническом имени элемента верхнего универсума).

Доказательство. Из теоремы 15 и предложения 57 следует доказу-доказуемость в **K** секвенции:

$V^C(S), V^D(Z) \vdash$

$\vdash \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q)$.

Следовательно, секвенция

$V^C(S), V^D(U) \vdash$

$\vdash \forall r \in \breve{\mathrm{Dom}}(\{\langle U, |Q| \rangle\})\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q)$

также доказуема в **K** .

Теорема 16 следует из доказуемости в **K** следующих секвенций:

$V^C(S), V^D(U) \vdash$

$\vdash \forall r \in \breve{\mathrm{Dom}}(\{\langle U, |Q| \rangle\})\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q)$;

$\vdash \neg\neg U \in \breve{\mathrm{Dom}}(\{\langle U, |Q| \rangle\})$.

Если выполняются условия :

$V^D(u), V^C(v)$,

$|Q| \subset'_{\overline{Q}\ \overline{Q}} \bigcup_{\mathrm{OR}(r)} [\![\mathrm{h}v \in \mathrm{h}\underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\breve{r}}}_R]\!]_Q$,

$|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) = u]\!]_Q$,

то будем говорить, что v является именем u.

Изтеорем 15 и 16 следует, что если выполняется условие $V^D(u)$ то $\hat{u}$ является именем u. Множество $\hat{u}$ будем называть каноническим именем u.

§ 5 Теорема об истинности в верхнем универсуме

В первом параграфе главы 3 второго тома были определены следующие предикаты:

${}^{Df}\vdash t \preceq u \sim \exists r, v\,(u = \langle r, v \rangle\ \&$

$\&\ t \in (\mathrm{Dom}(r) \times \mathrm{Dom}(v)) \cup (\mathrm{Dom}(v) \times \mathrm{Dom}(r)))$;

${}^{Df}\vdash f : u \preceq_n v \sim \mathrm{Fn}(f)\ \&\mathrm{Dom}(f) = \mathrm{Sc}(\mathrm{Sc}(n))\ \&$

$\&\ \neg\neg \langle 0, v \rangle \in f\ \&$

$\&\ \neg\neg \langle \mathrm{Sc}(n), u \rangle \in f\ \&$

$\&\ \forall l \in \mathrm{Sc}(n)\, \forall r, t\, (\langle l, r \rangle \in f\ \&\ \langle \mathrm{Sc}(l), t \rangle \in f \rightarrow \neg\neg\, t \preceq r)$;

${}^{Df}\vdash u \preceq_n v \sim \exists f\, (f : u \preceq_n v)$;

${}^{Df}\vdash u \check{\preceq} v \sim \exists n \in \omega\, (u \preceq_n v)$;

$\mathrm{TC}_{\preceq}(\mathrm{Z}) \leftrightharpoons \{t \in \mathrm{TC}(|\,Z\,|) \times \mathrm{TC}(|\,Z\,|) \mid \neg\neg\exists u \in Z\, (t \check{\preceq} u)\}$.

В первом параграфе главы 3 второго тома были приняты обозначения следующих операций:

${}_{\preceq}u \leftrightharpoons \{t \in \bigcup\bigcup\bigcup\bigcup u \times \bigcup\bigcup\bigcup\bigcup u \mid \neg\neg\, t \preceq u\}$;

${}_{\check{\preceq}}u \leftrightharpoons \mathrm{TC}_{\preceq}(\{u\})$;

$u' \leftrightharpoons \bigcup \mathrm{Dom}(\{u\})$;

$u'' \leftrightharpoons \bigcup \mathrm{Rg}(\{u\})$;

$\mathrm{Ev}(f, u, v) \leftrightharpoons$

$\leftrightharpoons \{t \in \mathrm{P}(\bigcup \mathrm{Rg}(v)) \mid \neg\neg\exists r \in \mathrm{Dom}(v)\, (t = v(r) \cap f(u,r))\}$;

$\mathrm{Ev}[f, u, v, R] \leftrightharpoons$

$\leftrightharpoons \{t \in \mathrm{P}(|\,R\,|) \mid \neg\neg\exists r \in \mathrm{Dom}(u)\, (t = u(r) \cap_{\overline{R}} \mathrm{Ev}(f, r, v))\}$;

$\mathrm{Ev}(f, u, v, R) \leftrightharpoons {}_{\overline{R}}\bigcup \mathrm{Ev}[f, u, v, R] \cap_{\overline{R}} \bigcup \mathrm{Ev}[f, v, u, R]$.

В первом параграфе главы 3 второго тома были определены следующие предикаты:

${}^{Df}\vdash \mathrm{ev}(f,u,R) \sim \mathrm{Fn}(f)\ \&\ \forall r \in {}_{\check{\preceq}}u\ \ \neg\neg \langle r, \mathrm{Ev}(f\,|\,{}_{\preceq}r, r', r'', R) \rangle \in f$

(f есть u – оценочная функция множества u в R);

${}^{Df}\vdash \mathrm{ev}[\, f,u,R\,] \sim \mathrm{ev}(f,u,R)\ \&\ \mathrm{Dom}(f) \subset' {}_{\check{\preceq}}u$.

В первом параграфе главы 3 второго тома были приняты обозначения следующих операций:

$$\hat{\mathrm{Ev}}(u,R) \rightleftharpoons \{t \in {}_{\precsim}u \times \mathrm{P}(|R|) \mid \neg\neg\exists r \in {}_{\precsim}u\, \exists f\, (\mathrm{ev}(f,u,R)\ \&\ t = $$
$$= \langle r, \mathrm{Ev}(f \mid {}_{\preceq}r, r', r'', R) \rangle)\}$$

($\hat{\mathrm{Ev}}(u,R)$ есть оценочная функция множества u в R);

$$[\![u = v]\!]_R \rightleftharpoons \mathrm{Ev}(\hat{\mathrm{Ev}}(\langle u, v \rangle, R) \mid {}_{\preceq}\langle u, v \rangle, u, v, R)$$

($[\![u = v]\!]_R$ есть оценка формулы $u = v$ в R);

$$[\![u \in v]\!]_R \rightleftharpoons \bigcup_{r \in \mathrm{Dom}(v)} (v(r) \cap [\![u = r]\!]_R)$$

($[\![u \in v]\!]_R$ есть оценка формулы $u \in v$ в R).

$$\mathrm{Ev}_{=}(u,v,R) \rightleftharpoons$$
$$\rightleftharpoons \{t \in \mathrm{P}(\bigcup \mathrm{Rg}(v)) \mid \neg\neg\exists r \in \mathrm{Dom}(v)\, (t = v(r) \cap [\![u = r]\!]_R)\};$$
$$\mathrm{Ev}_{\notin}(u,v,R) \rightleftharpoons$$
$$\rightleftharpoons \{t \in \mathrm{P}(|R|) \mid \neg\neg\exists r \in \mathrm{Dom}(u)\, (t = u(r) \cap \neg_R [\![r \in v]\!]_R)\};$$
$$\mathrm{Ev}_{=}[u,v,R] \rightleftharpoons \neg_R \bigcup \mathrm{Ev}_{\notin}(u,v,R) \cap \neg_R \bigcup \mathrm{Ev}_{\notin}(v,u,R).$$

Определение внутренних R – операций

$\underline{\mathrm{Ev}}_R(f, u, v)$, $\underline{\mathrm{Ev}}_R[f, u, v, S]$, $\underline{\mathrm{Ev}}_R(f, u, v, S)$, $\underline{\hat{\mathrm{Ev}}}_R(u, S)$,
$\underline{[\![u = v]\!]_S}_R$, $\underline{[\![u \in v]\!]_S}_R$, $\underline{\mathrm{Ev}_{=}}_R(u,v,S)$, $\underline{\mathrm{Ev}_{\notin}}_R(u,v,S)$, $\underline{\mathrm{Ev}_{=}}_R[u,v,S]$
предоставляется читателю.

Предложение 58 (**K**)

$$V^C(u), V^C(S) \vdash [\![V^{\underline{O}_R(S)}(u)]\!]_R = \neg_R \neg_R \bigcup_{\mathrm{OR}(r)} [\![u \in \underline{\mathrm{V}^{\underline{O}_R(S)}_{\check{r}}}_R]\!]_R .$$

Доказательство.

$$V^C(u), V^C(S) \vdash [\![V^{\underline{O}_R(S)}(u)]\!]_R = [\![\neg\neg\exists r(\mathrm{OR}(r)\ \&\ u \in \mathrm{V}^{\underline{O}_R(S)}_{r})]\!]_R =$$
$$= \neg_R \neg_R [\![\exists r(\mathrm{OR}(r)\ \&\ u \in \mathrm{V}^{\underline{O}_R(S)}_{r})]\!]_R =$$
$$= \neg_R \neg_R \bigcup_{V^C(r)} [\![\mathrm{OR}(r)\ \&\ u \in \mathrm{V}^{\underline{O}_R(S)}_{r}]\!]_R =$$
$$= \neg_R \neg_R \bigcup_{V^C(r)} ([\![\mathrm{OR}(r)]\!]_R \cap [\![u \in \mathrm{V}^{\underline{O}_R(S)}_{r}]\!]_R) =$$
$$= \neg_R \neg_R \bigcup_{V^C(r)} ([\![\mathrm{OR}(r)]\!]_R \cap \neg_R \neg_R [\![u \in \mathrm{V}^{\underline{O}_R(S)}_{r}]\!]_R) =$$
$$= \neg_R \neg_R \bigcup_{V^C(r)} ([\![\mathrm{OR}(r)]\!]_R \cap \neg_R \neg_R [\![u \in \underline{\mathrm{V}^{\underline{O}_R(S)}_{r}}_R]\!]_R) =^{1)}$$
$$\neg_R \neg_R \bigcup_{\mathrm{OR}(r)} \neg_R \neg_R [\![u \in \underline{\mathrm{V}^{\underline{O}_R(S)}_{\check{r}}}_R]\!]_R =$$
$$\neg_R \neg_R \bigcup_{\mathrm{OR}(r)} [\![u \in \underline{\mathrm{V}^{\underline{O}_R(S)}_{\check{r}}}_R]\!]_R .$$

Равенство, отмеченное на вышеприведенном дереве символом [1], следует из теоремы 7

В настоящем параграфе принимаются следующие обозначения:

$Q \leftrightharpoons R \otimes'_{\Delta} S$;

$f \leftrightharpoons \{ t \in (|R| \times \Delta) \times |R| \mid \neg\neg \exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \}$;

$D \leftrightharpoons \mathrm{O}(R \otimes'_{\Delta} S)$.

Если $u \in \mathrm{O}(R)$, то примается следующее обозначение:

$\mathrm{h}u \leftrightharpoons f^{-1}{}''u$.

Если $V^{\mathrm{O}(R)}(u)$, то примается следующее обозначение:

$\mathrm{h}u \leftrightharpoons f^{-1}{}^{\frown}u$.

Также, для упрощения записи, из антецедентов всех секвенций настоящего параграфа опущена формула:

$|R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R$.

Предложение 59 (**K**)

$$V^C(u), V^C(S) \vdash f^{-1}{}'' [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R = \underset{Q}{-} \underset{Q}{-} \bigcup_{\mathrm{OR}(r)} [\![f^{-1}{}^{\frown}u \in f^{-1}{}^{\frown} \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_Q$$

(в сокращенном виде

$$V^C(u), V^C(S) \vdash \mathrm{h} [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R = \underset{Q}{-} \underset{Q}{-} \bigcup_{\mathrm{OR}(r)} [\![\mathrm{h}u \in \mathrm{h} \, \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_Q \,).$$

Доказательство.

$$V^C(u), V^C(S) \vdash f^{-1}{}'' [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R = f^{-1}{}'' \underset{R}{-} \underset{R}{-} \bigcup_{\mathrm{OR}(r)} [\![u \in \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_R =$$

$$= \underset{Q}{-} f^{-1}{}'' \underset{R}{-} \bigcup_{\mathrm{OR}(r)} [\![u \in \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_R =$$

$$= \underset{Q}{-} \underset{Q}{-} f^{-1}{}'' \bigcup_{\mathrm{OR}(r)} [\![u \in \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_R =$$

$$= \underset{Q}{-} \underset{Q}{-} \bigcup_{\mathrm{OR}(r)} f^{-1}{}'' [\![u \in \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_R =$$

$$= \underset{Q}{-} \underset{Q}{-} \bigcup_{\mathrm{OR}(r)} [\![f^{-1}{}^{\frown}u \in f^{-1}{}^{\frown} \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_Q .$$

Первое равенство в вышеприведенной цепи равенств следует из предложения 58.

Введем обозначение следующих операций:

$$^{Df} \vdash V_{R,S,Q}(u) \leftrightharpoons \underset{Q}{-} \underset{Q}{-} \bigcup_{\mathrm{OR}(r)} [\![u \in f^{-1}{}^{\frown} \underline{\mathrm{V}_{\check{r}}^{\underline{\mathrm{O}}_R(S)}}_R]\!]_Q ;$$

$$^{Df} \vdash V_{R,S,Q}(u, v) \leftrightharpoons V_{R,S,Q}(u) \cap V_{R,S,Q}(v) .$$

Предложение 60 (**K**)

$V^{C}(u), V^{C}(S) \vdash V_{R,S,Q}(f^{-1} \frown u) = f^{-1}\text{''}\, [\![V^{\mathrm{Q}_R(S)}(u)]\!]_R$.

Доказательство. Предложение 60 следует из предложения 59.

Предложение 61 (**K**)

$V^{C}(u), V^{C}(u) \vdash$

$\vdash |R| \subset' [\![\underline{[\![u = v]\!]}_{S\,R} = {}_{\overline{S}\,R} \bigcup \underline{\underline{\mathrm{Ev}_{\notin R}}(u,v,S)}_R \;\underline{\cap}_R\; {}_{\overline{S}\,R} \bigcup \underline{\underline{\mathrm{Ev}_{\notin R}}(v,u,S)}_R]\!]_R$.

Доказательство. Секвенция

$\vdash [\![u = v]\!]_R = {}_{\overline{R}} \bigcup \mathrm{Ev}_{\notin}(u,v,R) \cap {}_{\overline{R}} \bigcup \mathrm{Ev}_{\notin}(v,u,R)$

является доказуемой в **K**, а при ее доказательстве используются только та часть аксиом **K** , оценки которых равны $|R|$.

Из предложения 61 следует

Предложение 62 (**K**)

$V^{C}(u), V^{C}(u) \vdash$

$\vdash |Q| \subset' [\![\mathrm{h}\, \underline{[\![u = v]\!]}_{S\,R} = \mathrm{h}\; {}_{\overline{S}\,R} \bigcup \underline{\underline{\mathrm{Ev}_{\notin R}}(u,v,S)}_R \;\underline{\cap}_R\; {}_{\overline{S}\,R} \bigcup \underline{\underline{\mathrm{Ev}_{\notin R}}(v,u,S)}_R]\!]_Q$.

Предложение 63 (**K**)

$V^{C}(u), V^{C}(u) \vdash$

$\vdash |R| \subset' [\![\underline{[\![u \in v]\!]}_{S\,R} = \bigcup \underline{\underline{\mathrm{Ev}_{= R}}(u,v,S)}_R]\!]_R$.

Доказательство. Секвенция

$\vdash [\![u \in v]\!]_R = \bigcup \mathrm{Ev}_{=}(u,v,R)$

является доказуемой в **K**, а при ее доказательстве используются только та часть аксиом **K** , оценки которых равны $|R|$.

Из предложения 63 следует

Предложение 64 (**K**)

$V^{C}(u), V^{C}(u) \vdash$

$\vdash |Q| \subset' [\![\mathrm{h}\, \underline{[\![u \in v]\!]}_{S\,R} = \mathrm{h} \bigcup \underline{\underline{\mathrm{Ev}_{= R}}(u,v,S)}_R]\!]_Q$.

В настоящем параграфе принимается следующее обозначение:

$a \rightleftharpoons [\![\mathrm{Gn}(G, \mathrm{h}S, \mathrm{h}\, \check{\mathrm{P}}(|R|))]\!]_Q$.

Теорема 17 (**K**)

$V^{D}(G), V^{C}(X,Y,S) \vdash$

$\vdash [\![\mathrm{Gn}(G, \mathrm{h}S, \mathrm{h}\ \check{\mathrm{P}}(|\,R\,|))]\!]_{Q} \cap V_{R,S,Q}(\mathrm{h}X, \mathrm{h}Y) \cap [\![G \cap \mathrm{h}\,\underline{[\![X = Y]\!]_{SR}} \neq 0]\!]_{Q} =$

$= [\![\mathrm{Gn}(G, \mathrm{h}S, \mathrm{h}\ \check{\mathrm{P}}(|\,R\,|))]\!]_{Q} \cap V_{R,S,Q}(\mathrm{h}X, \mathrm{h}Y) \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}X, G) = \underline{\mathrm{In}}_{Q}(\mathrm{h}Y, G)]\!]_{Q}$.

(теорема об истинности в верхнем универсуме для предиката равенства) .

Доказательство. В ходе доказательства теоремы 17 используются следующие обозначения :

$A \rightleftharpoons \mathrm{TC}_{\preceq}(\langle X, Y \rangle) \cup \{\langle X, Y \rangle\}$;

$\mathfrak{R} \rightleftharpoons A_{\preceq}$

(операции $\mathrm{TC}_{\preceq}(u)$ и $A_{\preceq}$ определены в § 2 главы 3 второго тома);

$$Y \rightleftharpoons \{t \in \mathrm{TC}_{\preceq}(\langle X, Y \rangle) \cup \{\langle X, Y \rangle\} \mid \\ \mid a \cap V_{R,S,Q}(\mathrm{h}t', \mathrm{h}t'') \cap [\![G \cap \mathrm{h}\,\underline{[\![t' = t'']\!]_{SR}} \neq 0]\!]_{Q} = \\ = a \cap V_{R,S,Q}(\mathrm{h}t', \mathrm{h}t'') \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}t', G) = \underline{\mathrm{In}}_{Q}(\mathrm{h}t'', G)]\!]_{Q}\}$$

(операции u' и u'' определены в § 2 главы 3 второго тома).

Следующая секвенция доказана на схемах 2.5.1 – 2.5.14 (секвенция (7) схемы 2.5.14) с использованием предложений 62 и 64 :

$V^{D}(G), V^{C}(u,v,S), \mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash$

$\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap [\![G \cap \mathrm{h}\,\underline{[\![u = v]\!]_{SR}} \neq 0]\!]_{Q} =$

$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}u, G) = \underline{\mathrm{In}}_{Q}(\mathrm{h}v, G)]\!]_{Q}$.

Использование принципа $\preceq -$ индукции завершает доказательство теоремы 17.

Теорема 18 (**K**)

$V^{D}(G), V^{C}(X,Y,S) \vdash$

$\vdash [\![\mathrm{Gn}(G, \mathrm{h}S, \mathrm{h}\ \check{\mathrm{P}}(|\,R\,|))]\!]_{Q} \cap V_{R,S,Q}(\mathrm{h}X, \mathrm{h}Y) \cap [\![G \cap \mathrm{h}\,\underline{[\![X \notin Y]\!]_{SR}} \neq 0]\!]_{Q} =$

$= [\![\mathrm{Gn}(G, \mathrm{h}S, \mathrm{h}\ \check{\mathrm{P}}(|\,R\,|))]\!]_{Q} \cap V_{R,S,Q}(\mathrm{h}X, \mathrm{h}Y) \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}X, G) \notin \underline{\mathrm{In}}_{Q}(\mathrm{h}Y, G)]\!]_{Q}$.

(теорема об истинности в верхнем универсуме для отрицания предиката принадлежности).

Доказательство. Теорема 18 доказана на схемах 2.5.15 и 2.5.16 секвенция (9) схемы 2.5.16) с использованием теоремы 17 и секвенции (3) схемы 2.5.3. Равенство, отмеченное на схеме 2.5.15 символом [1], следует из теоремы 17.

Предложение 65

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, не содержащая переменной r, то в **K** доказуема следующая секвенция :

$$V^D(r), V^C(u,S) \vdash |Q| \subset' [\![\mathrm{h}\,\underline{[\![\varphi(u,\hat{r})]\!]_{SR}} \subset' \mathrm{h}\,\underline{[\![\exists v\,\varphi(u,v)]\!]_{SR}}]\!]_Q .$$

Доказательство. Предложение 65 доказано на схеме 2.5.17 (секвенция (1)) с использованием теоремы 15.

Предложение 66

Если $\varphi(u,r)$ – отрицательная формула **K** с двумя свободными переменными u и r, не содержащая переменной G, то в **K** доказуема следующая секвенция :

$$V^D(r,u), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset'$$
$$\subset' [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q .$$

Доказательство. Предложение 66 доказано на схеме 2.5.17 (секвенция (2)) с использованием теоремы 16.

Для отрицательной формулы $\varphi(u,v)$ системы **K** с двумя свободными переменными u и v, принимаем обозначение следующей операции :

$$\| \varphi(u,.) \|^S \rightleftharpoons \{r \in \mathrm{P}(|S|) \mid \neg\neg\exists\alpha(\mathrm{OR}(\alpha)\,\exists v \in \mathrm{V}_\alpha^{\mathrm{O}(S)}(r = [\![\varphi(u,v)]\!]_S))\} .$$

В **K** доказуема следующая секвенция :

$$\vdash \neg\neg\, r \in \| \varphi(u,.) \|^S \sim \neg\neg\exists\alpha(\mathrm{OR}(\alpha)\,\exists v \in \mathrm{V}_\alpha^{\mathrm{O}(S)}(r = [\![\varphi(u,v)]\!]_S)) .$$

Для операции $\| \varphi(u,.) \|^S$ мы можем определить внутреннюю R – операцию $\underline{\| \varphi(u,.) \|^S}_R$.

Предложение 67

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, то в **K** доказуема следующая секвенция :

$$V^C(u,r,S) \vdash \neg_R \neg_R [\![r \in \underline{\| \varphi(u,.) \|^S}_R]\!]_R =$$
$$= \neg_R \neg_R [\![\exists\alpha(\mathrm{OR}(\alpha)\,\exists v(\neg\neg\, v \in \mathrm{V}_\alpha^{\mathrm{O}(S)}\ \&\ r = [\![\varphi(u,v)]\!]_S))]\!]_R .$$

Предложение 68

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, то в **K** доказуема следующая секвенция :

$$V^C(u,r,S) \vdash_{\overline{R}\,\overline{R}} [\![r \in \underline{\| \varphi(u,.) \|^S_R}]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R]\!]_R \cap [\![r = \underline{[\![\varphi(u,v)]\!]_{SR}}]\!]_R) .$$

Доказательство. Предложение 68 доказано на схемах 2.5.17 и 2.5.18 (секвенция (3) схемы 2.5.18) с использованием предложения 67.

Из предложения 68 следует

Предложение 69

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, то в **K** доказуема следующая секвенция :

$$V^C(u,r,S) \vdash_{\overline{Q}\,\overline{Q}} [\![\mathrm{h}r \in \mathrm{h} \, \underline{\| \varphi(u,.) \|^S_R}]\!]_Q =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} ([\![\mathrm{h}v \in \mathrm{h} \, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R]\!]_Q \cap [\![\mathrm{h}r = \mathrm{h} \, \underline{[\![\varphi(u,v)]\!]_{SR}}]\!]_Q) .$$

Предложение 70

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, то в **K** доказуема следующая секвенция :

$$V^D(G), V^C(u,S) \vdash$$

$$\vdash [\![\underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h} \, \underline{[\![\exists v \, \varphi(u,v)]\!]_{SR}} \neq 0]\!]_Q =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(v)} (V_{R,S,Q}(\mathrm{h}v) \cap [\![\underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h} \, \underline{[\![\varphi(u,v)]\!]_{SR}} \neq 0]\!]_Q) .$$

Доказательство. Предложение 70 доказано на схемах 2.5.18 и 2.5.19 (секвенция (5) схемы 2.5.19) с использованием предложения 69. Равенство, отмеченное на схеме 2.5.18 символом [1)], следует из предложения 69 .

Предложение 71

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, и не содержащая переменных y, t, r и G, то в **K** доказуема следующая секвенция :

$$V^{C}(u,S),\ G=\mathrm{gn}(R\otimes'_{\Delta}S),\ \forall y,t\,(V^{C}(y,t)\to$$
$$\to V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}y,\underline{\mathrm{Rg}}_Q(G)),\underline{\mathrm{In}}_Q(\mathrm{h}t,\underline{\mathrm{Rg}}_Q(G)))]\!]_Q=$$
$$=V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\varphi(y,t)]\!]_{SR}}\neq 0]\!]_Q)\vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u)\cap[\![\exists r\,\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u,\underline{\mathrm{Rg}}_Q(G)),r)]\!]_Q\subset'$$
$$\subset'[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\exists v\,\varphi(u,v)]\!]_{SR}}\neq 0]\!]_Q.$$

Доказательство. Предложение 71 доказано на схемах 2.5.19 и 2.5.20 (секвенция (7) схемы 2.5.20) с использованием предложения 66.

Предложение 72

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v, и не содержащая переменных y, t, r и G, то в **K** доказуема следующая секвенция :

$$V^{C}(u),V^{D}(G),\ \forall y,t\,(V^{C}(y,t)\to$$
$$\to V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}y,\underline{\mathrm{Rg}}_Q(G)),\underline{\mathrm{In}}_Q(\mathrm{h}t,\underline{\mathrm{Rg}}_Q(G)))]\!]_Q=$$
$$=V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\varphi(y,t)]\!]_{SR}}\neq 0]\!]_Q)\vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u)\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\exists v\,\varphi(u,v)]\!]_{SR}}\neq 0]\!]_Q\subset'$$
$$\subset'_{\overline{Q}\ \overline{Q}}[\![\exists r\,\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u,\underline{\mathrm{Rg}}_Q(G)),r)]\!]_Q.$$

Доказательство. Предложение 72 доказано на схеме 2.5.21 (секвенция (8)) с использованием предложения 70.

Из предложений 71 и 72 следует

Предложение 73

Если $\varphi(u,v)$ – отрицательная формула **K** с двумя свободными переменными u и v , и не содержащая переменных y, t и G, то в **K** доказуема следующая секвенция :

$$V^{C}(u,S),\ G=\mathrm{gn}(R\otimes'_{\Delta}S),\ \forall y,t\,(V^{C}(y,t)\to$$
$$\to V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}y,\underline{\mathrm{Rg}}_Q(G)),\underline{\mathrm{In}}_Q(\mathrm{h}t,\underline{\mathrm{Rg}}_Q(G)))]\!]_Q=$$
$$=V_{R,S,Q}(\mathrm{h}y,\mathrm{h}t)\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\varphi(y,t)]\!]_{SR}}\neq 0]\!]_Q)\vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u)\cap[\![\neg\exists v\,\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u,\underline{\mathrm{Rg}}_Q(G)),v)]\!]_Q=$$
$$=V_{R,S,Q}(\mathrm{h}u)\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap\mathrm{h}\,\underline{[\![\neg\exists v\,\varphi(u,v)]\!]_{SR}}\neq 0]\!]_Q.$$

Аналогично предложению 73 доказывается

Предложение 74

Если $\varphi(u, \dots, v)$ – отрицательная формула **K**, где $u, \dots, v$ – список всех свободных переменных формулы $\varphi(u, \dots, v)$, не содержащей переменных y, t и G, то в **K** доказуема следующая секвенция :

$$V^{C}(u, \dots, S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y, t\, (V^{C}(y, t) \rightarrow$$
$$\rightarrow V_{R,S,Q}(\mathrm{h}y, \dots, \mathrm{h}t) \cap [\![\varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y, \underline{\mathrm{Rg}}_{Q}(G)), \dots, \underline{\mathrm{In}}_{Q}(\mathrm{h}t, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} =$$
$$= V_{R,S,Q}(\mathrm{h}y, \dots, \mathrm{h}t) \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![\varphi(y, \dots, t)]\!]_{SR}} \neq 0]\!]_{Q}) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u, \dots) \cap [\![\neg \exists v\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), \dots, v)]\!]_{Q} =$$
$$= V_{R,S,Q}(\mathrm{h}u, \dots) \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![\neg \exists v\, \varphi(u, \dots, v)]\!]_{SR}} \neq 0]\!]_{Q} .$$

Теорема 19

Если $\varphi(u, \dots, v)$ – отрицательная формула **K**, где $u, \dots, v$ – список всех свободных переменных формулы $\varphi(u, \dots, v)$, не содержащей переменной G, то в **K** доказуема следующая секвенция :

$$V^{C}(u, \dots, v, S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u, \dots, \mathrm{h}v) \cap [\![\varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), \dots, \underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} =$$
$$= V_{R,S,Q}(\mathrm{h}u, \dots, \mathrm{h}v) \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![\varphi(u, \dots, v)]\!]_{SR}} \neq 0]\!]_{Q})$$

(теорема об истинности в верхнем универсуме для отрицательных формул **K**).

Доказательство. Теорема 19 доказывается индукцией по длине отрицательной формулы **K** с использованием теорем 17, 18, и предложения 74.

Из теоремы 19 следует

Теорема 20

Если φ – отрицательная формула **K**, не содержащая свободных свободных переменных, то из доказуемости в **K** секвенции

$$\vdash |R| \subset' [\![0 \neq \underline{|S|}_{R} \subset' \underline{[\![\varphi]\!]_{SR}}]\!]_{R} ,$$

следует, что в **K** доказуема секвенция :

$$\vdash |Q| \subset' [\![\varphi]\!]_{Q} .$$

§ 6 Операции "промежуточного" универсума

Если выполняются условия :

$$V^{\mathrm{O}(R)}(v),\ |R| \subset' \underset{R}{-}\underset{R}{-} \bigcup_{\mathrm{OR}(r)} [\![v \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{r}}}_R]\!]_R ,$$

то будем говорить, что u является элементом "промежуточного" универсума.

Если для операции $\mathrm{F}(u)$ системы **K** можно определить внутреннюю S – операцию $\underline{\mathrm{F}}_S(u)$(возможность такого определения зависит от самой операции $\mathrm{F}(u)$ и от конкретного отношения S), то рассматривая операцию $\underline{\mathrm{F}}_S(u)$ как операцию **K**, можно определить внутреннюю R – операцию $\underline{\underline{\mathrm{F}}}_{S_R}(u)$.

Операции подобного рода будем называть R, S – операциями или операциями "промежуточного" универсума.

В § 3 главы 3 второго тома было введено обозначение следующей операции

$$\underline{\mathrm{P}}_S(u) \leftrightharpoons \{\langle r, [\![r \subset' u]\!]_S \rangle \mid r \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))\}$$

($\underline{\mathrm{P}}_S(u)$ есть внутренняя R – операция множества-степени u в $\mathrm{O}(S)$ – универсуме).

Введем обозначения следующей операции:

$$\underline{\underline{\mathrm{P}}}_{S_R}(u) \leftrightharpoons \{\langle r, [\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(r = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \rangle \mid$$
$$\mid r \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)) \underline{\times}_R \underline{\mathrm{P}}_R(\underline{|S|}_R))\} .$$

Предложение 75 (**K**)

$$V^C(t), V^C(u)\, V^C(S) \vdash \underset{R}{-}\underset{R}{-} [\![t \in \underline{\underline{\mathrm{P}}}_{S_R}(u)]\!]_R =$$
$$= \underset{R}{-}\underset{R}{-} [\![\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R .$$

Доказательство. Предложение 75 доказано на схемах 2.6.1 – 2.6.3 (секвенция (1) схемы 2.6.3).

Предложение 76 (**K**)

$$V^C(t), V^C(v), V^C(u)\, V^C(S) \vdash \underset{R}{-}\underset{R}{-} [\![\underline{\langle t, v \rangle}_R \in \underline{\underline{\mathrm{P}}}_{S_R}(u)]\!]_R =$$
$$= \underset{R}{-}\underset{R}{-} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap$$
$$\cap [\![t = y]\!]_R \cap [\![v = \underline{[\![y \subset' u]\!]_{S_R}}]\!]_R) .$$

Доказательство. Предложение 76 доказано на схеме 2.6.3 (секвен-
(секвенция (2) схемы 2.6.3) с использованием предложения 75.

Из предложения 76 следует

Предложение 77 (**K**)

$$V^{C}(u)\, V^{C}(S) \vdash |R| \subset' [\![\underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u)) = \underline{\mathrm{PF}}_{R}(\underline{\mathrm{Dom}}_{R}(u) \underline{\times}_{R} \underline{\mathrm{O}}_{R}(S))]\!]_{R} .$$

Из предложения 76 следует

Предложение 78 (**K**)

$$V^{C}(u) \vdash |R| \subset' [\![V^{\underline{\mathrm{O}}_{R}(S)}(u) \to V^{\underline{\mathrm{O}}_{R}(S)}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{R} .$$

Предложение 79 (**K**)

$$V^{C}(u), V^{C}(t) \vdash$$

$$\vdash [\![V^{\underline{\mathrm{O}}_{R}(S)}(u) \,\&\, V^{\underline{\mathrm{O}}_{R}(S)}(t) \,\&\, \mathrm{Tv}(S) \to |S| \subset' [\![t \in \underline{\underline{\mathrm{P}}}_{SR}(u) \sim t \subset' u]\!]_{S}]\!]_{R} .$$

Доказательство. Предложение 79 следует из того, что секвенция

$$V^{\mathrm{O}(S)}(u) \,\&\, V^{\mathrm{O}(S)}(t) \,\&\, \mathrm{Tv}(S) \to |S| \subset' [\![t \in \underline{\mathrm{P}}_{S}(u) \sim t \subset' u]\!]_{S}$$

доказуема в **K** с использованием только тех аксиом **K**, оценки которых равны $|S|$.

Предложение 79 может быть доказано и прямым вычислением оценки

$$[\![V^{\underline{\mathrm{O}}_{R}(S)}(u) \,\&\, V^{\underline{\mathrm{O}}_{R}(S)}(t) \,\&\, \mathrm{Tv}(S) \to |S| \subset' [\![t \in \underline{\underline{\mathrm{P}}}_{SR}(u) \sim t \subset' u]\!]_{S}]\!]_{R}$$

с использованием предложения 75. Такой способ доказательства является великолепным упражнением для читателя.

В настоящем параграфе принимаются следующие обозначения:

$Q \leftrightharpoons R \otimes'_{\Delta} S$;

$f \leftrightharpoons \{ t \in (|R| \times \Delta) \times |R| \mid \neg\neg \exists p \in |R| \, \exists \sigma \in \Delta \, (t = \langle \langle p, \sigma \rangle, p \rangle) \}$;

$D \leftrightharpoons \mathrm{O}(R \otimes'_{\Delta} S)$;

$G \leftrightharpoons \mathrm{gn}(R \otimes'_{\Delta} S)$.

Если $u \in \mathrm{O}(R)$, то примается следующее обозначение:

$\mathrm{h}u \leftrightharpoons f^{-1}{}'' u$.

Если $V^{\mathrm{O}(R)}(u)$, то примается следующее обозначение:

$\mathrm{h}u \leftrightharpoons f^{-1} {}^{\frown} u$.

Также, для упрощения записи, из антецедентов всех секвенций настоящего параграфа опущена формула:

$| R | \subset' [\![\mathrm{Tv}(S) \; \& \; | S | = \check{\Delta}]\!]_R$.

Предложение 80 (**K**)

$V^C(u,S), V^D(t) \vdash$

$\vdash \overline{}_Q \overline{}_Q [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\,\underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$

$= \overline{}_Q \overline{}_Q \bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$

$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}} \neq 0]\!]_Q \cap$

$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$.

Доказательство. Предложение 80 доказано на схеме 2.6.4 (секвенция (1)).

Предложение 81 (**K**)

$V^C(u,S) \vdash$

$\vdash V_{R,S,Q}(\mathrm{h}u) \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\,\underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q$.

Доказательство. Предложение 81 доказано на схемах 2.6.5 – 2.6.8 (секвенция (5) схемы 2.6.8) с использованием предложения 80 и теоремы об истинности в верхнем универсуме формулы $u \subset' v$ (частного случая теоремы 19), из которой следует равенство, отмеченное на схеме 2.6.5 символом [1].

Введем обозначение следующей операции:

$u_{\cap Z\, R,S,Q,G} \rightleftharpoons \{ \langle r, \bigcup_{V^{\mathrm{O}(R)}(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_S \cap$

$\cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$

$\cap \Big[\!\Big[r = \langle v, \underline{u(v)}_R \, \underline{\cap}_R \, \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \Big]\!\Big]_R) \rangle |$

$r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{P}}_R(\underline{\bigcup \underline{\mathrm{Rg}}_R(u)}_R)) \}$.

Для упрощения записи буквы R, S, Q, G в обозначении операции $u_{\cap Z\, R,S,Q,G}$ будем опускать.

Предложение 82 (**K**)

$V^C(t), V^C(u), V^C(S) \vdash [\![t \in u_{\cap Z}]\!]_R =$

$= \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$

$\cap \Big[\!\Big[t = \langle v, \underline{u(v)}_R \, \underline{\cap}_R \, \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \Big]\!\Big]_R)$.

Доказательство. Предложение 82 доказано на схемах 2.6.9 – 2.6.11 (секвенция (3) схемы 2.6.11)

Предложение 83 (**K**)

$$V^C(r,t), V^C(u), V^C(S) \vdash [\![\underline{\langle r, t \rangle}_R \in u_{\cap Z}]\!]_R =$$

$$= \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$

$$\cap [\![r = v]\!]_R \cap \left[\!\left[t = \underline{u(v)}_R \,\underline{\cap}_R \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \right]\!\right]_R).$$

Доказательство. Предложение 83 доказано на схеме 2.6.11 (секвенция (4)) с использованием предложения 82.

Предложение 84 (**K**)

$$V^C(r,u,S), V^D(Z) \vdash \forall v ([\![r = v]\!]_R \subset'$$

$$\subset' \left\| \underline{u(r)}_R \,\underline{\cap}_R \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \right.$$

$$\left. = \underline{u(v)}_R \,\underline{\cap}_R \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \right\|_R).$$

Доказательство. Предложение 84 доказано на схеме 2.6.12 (секвенция (5)) с использованием следствия предложения 44, отмеченного на схеме 2.6.12 символом [1)].

Предложение 85 (**K**)

$$V^C(r,t,u,S), V^D(Z) \vdash [\![\underline{\langle r, t \rangle}_R \in u_{\cap Z}]\!]_R =$$

$$= [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$

$$\cap \left[\!\left[t = \underline{u(r)}_R \,\underline{\cap}_R \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \right]\!\right]_R.$$

Доказательство. Предложение 85 доказано на схемах 2.6.12 и 2.6.13 (секвенция (6) схемы 2.6.13) с использованием предложений 83 и 84.

Предложение 86 (**K**)

$$V^C(r,u,S), V^D(Z) \vdash$$

$$\vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \subset'$$

$$\subset' \left[\!\left[\underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R \in u_{\cap Z} \right]\!\right]_R.$$

Доказательство. Предложение 86 доказано на схемах 2.6.13 и 2.6.14 (секвенция (7) схемы 2.6.14) с использованием предложения 82.

Предложение 87 (**K**)

$V^C(r,u,S), V^D(Z) \vdash$

$\vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \subset'$

$\subset' [\![\underline{u_{\cap Z}(r)}_R = \underline{u(r)}_R \, \underline{\cap}_R \, \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S)]\!]_R$.

Доказательство. Предложение 87 доказано на схеме 2.6.14 (секвенция (8)) с использованием предложений 85 и 86.

Предложение 88 (**K**)

$V^C(u,S), V^D(t,Z) \vdash$

$\vdash \overline{}_Q \overline{}_Q [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$

$= \overline{}_Q \overline{}_Q \bigcup\limits_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$

$\cap\, \mathrm{h}\, \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$

$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u_{\cap Z}(r)}_R \neq 0]\!]_Q \cap$

$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$.

Доказательство. Предложение 88 доказано на схемах 2.6.14 и 2.6.15 (секвенция (10) схемы 2.6.15) с использованием предложения 85.

Теорема 21 (**K**)

$V^C(u,S), V^D(Z) \vdash$

$\vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G)) = Z \, \underline{\cap}_Q \, \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q$.

(теорема об интерпретации $u_{\cap Z}$).

Доказательство. Теорема 21 доказана на схемах 2.6.16 – 2.6.18 (секвенция (11) схемы 2.6.18) с использованием предложений 87, 88 и предложения 47 (§ 4) , отмеченного на схеме 2.6.17 символом [2)].

Предложение 89 (**K**)

$V^C(u,S), V^D(Z) \vdash |Q| \subset' \overline{}_Q \overline{}_Q [\![\mathrm{h}\, u_{\cap Z} \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_Q$.

Доказательство. Предложение 89 доказано на схеме 2.6.19 (секвенция (12)).

Предложение 90 (**K**)

$V^C(u,S), V^D(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, [\![\underline{u_{\cap Z} \subset' u}]\!]_{SR} \neq 0]\!]_Q$

Доказательство. Предложение 90 доказано на схеме 2.6.19 (секвенция (13)) с использованием теоремы 19, отмеченной на схеме 2.6.19 символом [1].

Предложение 91 (**K**)

$$V^C(u,S), V^D(Z) \vdash [\![Z \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset'$$
$$\subset' [\![Z = \underline{\mathrm{In}}_Q(\mathrm{h}\ u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

Доказательство. Предложение 91 доказано на схеме 2.6.20 (секвенция (14)) с использованием теоремы 21, отмеченной на схеме 2.6.19 символом [2].

Предложение 92 (**K**)

$$V^C(u,S), V^D(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset'$$
$$\subset' \overline{\underline{\ }_Q}\,\overline{\underline{\ }_Q}[\![Z \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{S\,R}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

Доказательство. Предложение 92 доказано на схеме 2.6.20 (секвенция (15)) с использованием предложений 89, 90, 91 и предложения 80, отмеченного на схеме 2.6.20 символом [3].

Предложение 93 (**K**)

$$V^C(u,S) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset' [\![\underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{S\,R}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

Доказательство. Предложение 93 доказано на схеме 2.6.20 (секвенция (22)) с использованием предложения 92.

Из предложений 81 и 93 следует

Теорема 22 (**K**)

$$V^C(u,S) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset' [\![\underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))) = \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{S\,R}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

(теорема об интерпретации R,S – операции $\underline{\underline{\mathrm{P}}}_{S\,R}(u)$).

Мы можем определить R,S – операцию $\underline{\underline{\mathrm{V}}}_{u_{S\,R}}$ для S – операции $\underline{\mathrm{V}}_{u_S}$. Детали определения оставляются читателю.

Следующие два предложения следуют из определения $\underline{\underline{\mathrm{V}}}_{u_{S\,R}}$.

Предложение 94 (**K**)

$$V^C(r), V^C(u), V^C(S) \vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R \subset'$$
$$\subset' [\![\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{r\ S\,R}) \underline{\times}_R \underline{\mathrm{O}}_R(S)) \subset' \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\ S\,R})]\!]_R .$$

Предложение 95 (**K**)

$$V^C(u), V^C(r), V^C(v), V^C(S) \vdash$$

$$\vdash [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R \cap [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![v \in \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})]\!]_R \subset'$$

$$\subset' [\![\underline{u(r)}_R \,\underline{\cap}_R\, \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{\,SR}]\!]_{SR}} \subset' \underline{\underline{\mathrm{V}_u}}_{\,SR}\underline{(v)}_R]\!]_R .$$

Предложение 96 (**K**)

$$V^C(u), V^C(S),$$

$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v) \subset'$$

$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}_v}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$

$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

Доказательство. Предложение 95 доказано на схемах 2.6.22 – 2.6.30 (секвенция (7) схемы 2.6.30) с использованием теоремы 22 и предложений 94 и 95. Включение, отмеченное на схеме 2.6.24 символом [1)] следует из теоремы 22.

Из предложения 96 следует

Предложение 97 (**K**)

$$V^C(u), V^C(S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$

$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

Аналогично предложению 97 доказывается

Предложение 98 (**K**)

$$V^C(u), V^C(S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$

$$\subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q .$$

Из предложений 97 и 98 следует

Теорема 23 (**K**)

$$V^C(u), V^C(S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$

$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q = \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q .$$

(теорема об интерпретации R,S – операции $\underline{\underline{\mathrm{V}_u}}_{\,SR}$).

Для других R,S – операций могут быть доказаны теоремы аналогичные теоремам 22 и 23.

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash Z''[\![u = v]\!]_{R} =$$

$$= Z''\big(-_{R} \bigcup_{r\in \mathrm{Dom}(u)} (u(r)\cap [\![r \in v]\!]_{R}) \cap -_{R} \bigcup_{r\in \mathrm{Dom}(v)} (v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= Z'' -_{R}\big(-_{R} \bigcup_{r\in \mathrm{Dom}(u)} (u(r)\cap [\![r \in v]\!]_{R}) \supset_{R} -_{R} -_{R} \bigcup_{r\in \mathrm{Dom}(v)} (v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q} Z''\big(-_{R} \bigcup_{r\in \mathrm{Dom}(u)} (u(r)\cap [\![r \in v]\!]_{R}) \supset_{R} -_{R} -_{R} \bigcup_{r\in \mathrm{Dom}(v)} (v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(Z'' -_{R} \bigcup_{r\in \mathrm{Dom}(u)} (u(r)\cap [\![r \in v]\!]_{R}) \supset_{Q} Z'' -_{R} -_{R} \bigcup_{r\in \mathrm{Dom}(v)} (v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(Z'' -_{R} \bigcup_{r\in \mathrm{Dom}(u)} -_{R} -_{R}(u(r)\cap [\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} Z'' -_{R} -_{R} \bigcup_{r\in \mathrm{Dom}(v)} -_{R} -_{R}(v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} Z'' \bigcup_{r\in \mathrm{Dom}(u)} -_{R} -_{R}(u(r)\cap [\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} -_{Q} Z'' -_{R} \bigcup_{r\in \mathrm{Dom}(v)} -_{R} -_{R}(v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} Z'' \bigcup_{r\in \mathrm{Dom}(u)} -_{R} -_{R}(u(r)\cap [\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} Z'' \bigcup_{r\in \mathrm{Dom}(v)} -_{R} -_{R}(v(r)\cap [\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} Z'' \bigcup_{r\in \mathrm{Dom}(u)} -_{R}(u(r) \supset_{R} -_{R}[\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} Z'' \bigcup_{r\in \mathrm{Dom}(v)} -_{R}(v(r) \supset_{R} -_{R}[\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} \bigcup_{r\in \mathrm{Dom}(u)} Z'' -_{R}(u(r) \supset_{R} -_{R}[\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} \bigcup_{r\in \mathrm{Dom}(v)} Z'' -_{R}(v(r) \supset_{R} -_{R}[\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} \bigcup_{r\in \mathrm{Dom}(u)} -_{Q} Z''(u(r) \supset_{R} -_{R}[\![r \in v]\!]_{R}) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} \bigcup_{r\in \mathrm{Dom}(v)} -_{Q} Z''(v(r) \supset_{R} -_{R}[\![r \in u]\!]_{R})\big) =$$

$$= -_{Q}\big(-_{Q} \bigcup_{r\in \mathrm{Dom}(u)} -_{Q}(Z''u(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(v)} (v(t)\cap [\![r = t]\!]_{R})) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} \bigcup_{r\in \mathrm{Dom}(v)} -_{Q}(Z''v(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(u)} (u(t)\cap [\![r = t]\!]_{R}))\big) =$$

$$= -_{Q}\big(-_{Q} \bigcup_{r\in \mathrm{Dom}(u)} -_{Q}(Z''u(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(v)} -_{R} -_{R}(v(t)\cap [\![r = t]\!]_{R})) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} \bigcup_{r\in \mathrm{Dom}(v)} -_{Q}(Z''v(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(u)} -_{R} -_{R}(u(t)\cap [\![r = t]\!]_{R}))\big) =$$

$$= -_{Q}\big(-_{Q} \bigcup_{r\in \mathrm{Dom}(u)} -_{Q}(Z''u(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(v)} -_{R}(v(t) \supset_{R} -_{R}[\![r = t]\!]_{R})) \supset_{Q}$$

$$\supset_{Q} -_{Q} -_{Q} \bigcup_{r\in \mathrm{Dom}(v)} -_{Q}(Z''v(r) \supset_{Q} Z'' -_{R} \bigcup_{t\in \mathrm{Dom}(u)} -_{R}(u(t) \supset_{R} -_{R}[\![r = t]\!]_{R}))\big) =$$

$$
\begin{aligned}
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} Z'' \bigcup_{t\in \mathrm{Dom}(v)} \underset{R}{-}(v(t) \underset{R}{\supset} \underset{R}{-}\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} Z'' \bigcup_{t\in \mathrm{Dom}(u)} \underset{R}{-}(u(t) \underset{R}{\supset} \underset{R}{-} \llbracket r = t\rrbracket_R))) = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} Z'' \underset{R}{-}(v(t) \underset{R}{\supset} \underset{R}{-}\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} Z'' \underset{R}{-}(u(t) \underset{R}{\supset} \underset{R}{-} \llbracket r = t\rrbracket_R))) = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-} Z''(v(t) \underset{R}{\supset} \underset{R}{-}\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-} Z''(u(t) \underset{R}{\supset} \underset{R}{-} \llbracket r = t\rrbracket_R))) = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(t) \underset{Q}{\supset} Z'' \underset{R}{-}\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} Z'' \underset{R}{-} \llbracket r = t\rrbracket_R))) = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(t) \underset{Q}{\supset} \underset{Q}{-} Z''\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-} Z''\llbracket r = t\rrbracket_R)))
\end{aligned}
$$

$$
\begin{aligned}
&V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash Z''\llbracket u = v\rrbracket_R = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(t) \underset{Q}{\supset} \underset{Q}{-} Z''\llbracket r = t\rrbracket_R)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-} Z''\llbracket r = t\rrbracket_R))) ;
\end{aligned}
$$

$$
\begin{aligned}
&\qquad \mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash \\
&\vdash \forall r \in \mathrm{Dom}(u)\, \forall t \in \mathrm{Dom}(v)\, (Z''\llbracket r = t\rrbracket_R = \llbracket Z^\frown r = Z^\frown t\rrbracket_Q)\ \& \\
&\&\ \forall r \in \mathrm{Dom}(v)\, \forall t \in \mathrm{Dom}(u)\, (Z''\llbracket r = t\rrbracket_R = \llbracket Z^\frown r = Z^\frown t\rrbracket_Q)
\end{aligned}
$$

$$
\begin{aligned}
&V^{\mathrm{O}(R)}(u,v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R,Q), \mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash Z''\llbracket u = v\rrbracket_R = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(t) \underset{Q}{\supset} \underset{Q}{-}\llbracket Z^\frown r = Z^\frown t\rrbracket_Q)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(r) \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-}\llbracket Z^\frown r = Z^\frown t\rrbracket_Q))) = \\
&= \underset{Q}{-}(\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(u)} \underset{Q}{-}(Z^\frown u\,(Z^\frown r) \underset{Q}{\supset} \\
&\qquad\qquad \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{-}(Z'' v(t) \underset{Q}{\supset} \underset{Q}{-}\llbracket Z^\frown r = Z^\frown t\rrbracket_Q)) \underset{Q}{\supset} \\
&\quad \underset{Q}{\supset} \underset{Q}{-}\underset{Q}{-} \bigcup_{r\in \mathrm{Dom}(v)} \underset{Q}{-}(Z^\frown v\,(Z^\frown r) \underset{Q}{\supset} \\
&\qquad\qquad \underset{Q}{\supset} \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-}\llbracket Z^\frown r = Z^\frown t\rrbracket_Q))) =
\end{aligned}
$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{\overline{\ \ }}(Z'' v(t) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = Z^\frown t]\!]_Q)) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{\overline{\ \ }}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = Z^\frown t]\!]_Q))) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{t\in \mathrm{Dom}(v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(Z^\frown t) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = Z^\frown t]\!]_Q)) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(Z^\frown t) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = Z^\frown t]\!]_Q))) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(r) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = r]\!]_Q)) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(r) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y = r]\!]_Q))) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }}(Z^\frown v(r) \cap [\![y = r]\!]_Q)) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }}(Z^\frown u(r) \cap [\![y = r]\!]_Q))) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown v)} (Z^\frown v(r) \cap [\![y = r]\!]_Q)) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }} \bigcup_{r\in \mathrm{Dom}(Z^\frown u)} (Z^\frown u(r) \cap [\![y = r]\!]_Q))) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}(Z^\frown u(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y \in Z^\frown v]\!]_Q) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}(Z^\frown v(y) \underset{Q}{\supset} \underset{Q}{\overline{\ \ }}[\![y \in Z^\frown u]\!]_Q)) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }}(Z^\frown u(y) \cap [\![y \in Z^\frown v]\!]_Q) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }}(Z^\frown v(y) \cap [\![y \in Z^\frown u]\!]_Q)) =$$

$$= \underset{Q}{\overline{\ \ }}(\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown u)} (Z^\frown u(y) \cap [\![y \in Z^\frown v]\!]_Q) \underset{Q}{\supset}$$

$$\underset{Q}{\supset} \underset{Q}{\overline{\ \ }}\,\underset{Q}{\overline{\ \ }} \bigcup_{y\in \mathrm{Dom}(Z^\frown v)} (Z^\frown v(y) \cap [\![y \in Z^\frown u]\!]_Q)) =$$

$$= \overline{\ }_{Q} \bigcup_{y \in \mathrm{Dom}(Z^{\frown} u)} (Z^{\frown} u(y) \cap [\![y \in Z^{\frown} v]\!]_{Q}) \cap$$

$$\cap \overline{\ }_{Q} \bigcup_{y \in \mathrm{Dom}(Z^{\frown} v)} (Z^{\frown} v(y) \cap [\![y \in Z^{\frown} u]\!]_{Q}) = [\![Z^{\frown} u = Z^{\frown} v]\!]_{Q}$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q), \mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash$$

$$\vdash Z\text{''} [\![u = v]\!]_{R} = [\![Z^{\frown} u = Z^{\frown} v]\!]_{Q} \qquad (1)$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash Z\text{''} [\![u \notin v]\!]_{R} =$$

$$= Z\text{''} \overline{\ }_{R} [\![u \in v]\!]_{R} =$$

$$= Z\text{''} \overline{\ }_{R} \bigcup_{r \in \mathrm{Dom}(v)} (v(r) \cap [\![u = r]\!]_{R}) =$$

$$= Z\text{''} \overline{\ }_{R} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{R} \overline{\ }_{R} (v(r) \cap [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} Z\text{''} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{R} \overline{\ }_{R} (v(r) \cap [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} Z\text{''} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{R} (v(r) \supset_{R} \overline{\ }_{R} [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} Z\text{''} \overline{\ }_{R} (v(r) \supset_{R} \overline{\ }_{R} [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{Q} Z\text{''} (v(r) \supset_{R} \overline{\ }_{R} [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{Q} (Z\text{''} v(r) \supset_{Q} Z\text{''} \overline{\ }_{R} [\![u = r]\!]_{R}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{Q} (Z\text{''} v(r) \supset_{Q} \overline{\ }_{Q} Z\text{''} [\![u = r]\!]_{R}) = \text{[1)]}$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{Q} (Z\text{''} v(r) \supset_{Q} \overline{\ }_{Q} [\![Z^{\frown} u = Z^{\frown} r]\!]_{Q}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(v)} \overline{\ }_{Q} (Z^{\frown} v(Z^{\frown} r) \supset_{Q} \overline{\ }_{Q} [\![Z^{\frown} u = Z^{\frown} r]\!]_{Q}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(Z^{\frown} v)} \overline{\ }_{Q} (Z^{\frown} v(t) \supset_{Q} \overline{\ }_{Q} [\![Z^{\frown} u = t]\!]_{Q}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(Z^{\frown} v)} \overline{\ }_{Q} \overline{\ }_{Q} (Z^{\frown} v(t) \cap [\![Z^{\frown} u = t]\!]_{Q}) =$$

$$= \overline{\ }_{Q} \bigcup_{r \in \mathrm{Dom}(Z^{\frown} v)} (Z^{\frown} v(t) \cap [\![Z^{\frown} u = t]\!]_{Q}) =$$

$$= \overline{\ }_{Q} [\![Z^{\frown} u \in Z^{\frown} v]\!]_{R} = [\![Z^{\frown} u \notin Z^{\frown} v]\!]_{Q}$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$$

$$\vdash Z\text{''} [\![u \notin v]\!]_{R} = [\![Z^{\frown} u \notin Z^{\frown} v]\!]_{Q} \qquad (2)$$

Схема 2.1.5

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$$
$$\vdash Z'' [\![\neg\exists t \in u\, \varphi(t,v)]\!]_R =$$
$$= Z'' \underset{R}{-}[\![\exists t \in u\, \varphi(t,v)]\!]_R =$$
$$= Z'' \underset{R}{-} \bigcup_{t\in \mathrm{Dom}(u)} (u(t) \cap [\![\varphi(t,v)]\!]_R) =$$
$$= Z'' \underset{R}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{R}{-}\underset{R}{-}(u(t) \cap [\![\varphi(t,v)]\!]_R) =$$
$$= Z'' \underset{R}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{R}{-}\underset{R}{-}(u(t) \cap [\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} Z'' \bigcup_{t\in \mathrm{Dom}(u)} \underset{R}{-}\underset{R}{-}(u(t) \cap [\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} Z'' \bigcup_{t\in \mathrm{Dom}(u)} \underset{R}{-}(u(t) \underset{R}{\supset} \underset{R}{-}[\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} Z'' \underset{R}{-}(u(t) \underset{R}{\supset} \underset{R}{-}[\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-} Z'' (u(t) \underset{R}{\supset} \underset{R}{-}[\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} Z'' \underset{R}{-}[\![\varphi(t,v)]\!]_R) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-} Z'' [\![\varphi(t,v)]\!]_R)$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q) \vdash$$
$$\vdash Z'' [\![\neg\exists t \in u\, \varphi(t,v)]\!]_R =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-} Z'' [\![\varphi(t,v)]\!]_R)\,;$$
$$\forall t,r\, (V^{\mathrm{O}(R)}(t)\ \&\ V^{\mathrm{O}(R)}(r) \to Z'' [\![\varphi(t,r)]\!]_R = [\![\varphi(Z^\frown t, Z^\frown r)]\!]_Q),$$
$$V^{\mathrm{O}(R)}(u) \vdash \forall t \in \mathrm{Dom}(u)\, (Z'' [\![\varphi(t,v)]\!]_R = [\![\varphi(Z^\frown t, Z^\frown v)]\!]_Q)$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z,R,Q), \mathrm{Tv}(R), \mathrm{Tv}(Q),$$
$$\forall t,r\, (V^{\mathrm{O}(R)}(t)\ \&\ V^{\mathrm{O}(R)}(r) \to Z'' [\![\varphi(t,r)]\!]_R = [\![\varphi(Z^\frown t, Z^\frown r)]\!]_Q) \vdash$$
$$\vdash Z'' [\![\neg\exists t \in u\, \varphi(t,v)]\!]_R =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}(Z'' u(t) \underset{Q}{\supset} \underset{Q}{-}[\![\varphi(Z^\frown t, Z^\frown v)]\!]_Q) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} \underset{Q}{-}\underset{Q}{-}(Z'' u(t) \cap [\![\varphi(Z^\frown t, Z^\frown v)]\!]_Q) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} (Z'' u(t) \cap [\![\varphi(Z^\frown t, Z^\frown v)]\!]_Q) =$$
$$= \underset{Q}{-} \bigcup_{t\in \mathrm{Dom}(u)} (Z^\frown u\, (Z^\frown t) \cap [\![\varphi(Z^\frown t, Z^\frown v)]\!]_Q) =$$

Схема 2.1.6

$$= \frac{}{Q} \bigcup_{r \in \mathrm{Dom}(Z^\frown u)} (\, Z^\frown u\,(\,r\,) \cap [\![\varphi(r, Z^\frown v)]\!]_Q\,) =$$

$$= \frac{}{Q} [\![\exists r \in Z^\frown u \;\; \varphi(r, Z^\frown v)]\!]_Q =$$

$$= [\![\neg \exists r \in Z^\frown u \;\; \varphi(r, Z^\frown v)]\!]_Q$$

$$V^{\mathrm{O}(R)}(u), V^{\mathrm{O}(R)}(v), \mathrm{hm}(Z, R, Q), \mathrm{Tv}(R), \mathrm{Tv}(Q),$$

$$\forall t, r\,(V^{\mathrm{O}(R)}(\,t\,)\,\&\, V^{\mathrm{O}(R)}(\,r\,) \to Z\,"[\![\varphi(t, r)]\!]_R = [\![\varphi(Z^\frown t, Z^\frown r)]\!]_Q) \vdash$$

$$\vdash Z\,"[\![\neg \exists t \in u\, \varphi(t, v)]\!]_R = [\![\neg \exists r \in Z^\frown u \;\; \varphi(r, Z^\frown v)]\!]_Q \qquad (3)$$

Схема 2.2.1

$$
\begin{array}{l}
\langle t, r\rangle \in f, \langle t, y\rangle \in f, \mathrm{Fn}(f) \vdash r = y\,; \\
\qquad\qquad\qquad\qquad r = y, y \in v \vdash r \in v \\
\hline
\langle t, r\rangle \in f, \langle t, y\rangle \in f, \mathrm{Fn}(f), y \in v \vdash r \in v \\
\hline
\langle t, r\rangle \in f, \langle t, y\rangle \in f, \mathrm{Fn}(f), y \in v, r \notin v \vdash \\
\hline
\neg\neg\exists y \in v\,(\langle t, y\rangle \in f), \langle t, r\rangle \in f, \mathrm{Fn}(f), r \notin v \vdash \\
\qquad t \in f^{-1}{}''v \vdash \neg\neg\exists y \in v\,(\langle t, y\rangle \in f) \\
\hline
\qquad t \in f^{-1}{}''v, \langle t, r\rangle \in f, \mathrm{Fn}(f), r \notin v \vdash \\
\hline
\qquad\qquad \langle t, r\rangle \in f, \mathrm{Fn}(f), r \notin v \vdash t \notin f^{-1}{}''v
\end{array}
\qquad (1)
$$

$$
\begin{array}{l}
\langle t, r\rangle \in f, r \in u \vdash \neg\neg\, t \in f^{-1}{}''u\,; \\
\qquad\qquad \neg\neg\, t \in f^{-1}{}''u, t \in [p]_Q \vdash \neg\neg\, t \in [p]_Q \cap f^{-1}{}''u \\
\hline
\langle t, r\rangle \in f, r \in u, t \in [p]_Q \vdash \neg\neg\, t \in [p]_Q \cap f^{-1}{}''u\,; \qquad (1) \\
\hline
\langle t, r\rangle \in f, r \in u, t \in [p]_Q, \mathrm{Fn}(f), r \notin v \vdash \\
\quad \vdash \neg\neg\, t \in [p]_Q \cap f^{-1}{}''u \;\&\; t \notin f^{-1}{}''v\,; \\
\quad \neg\neg\, t \in [p]_Q \cap f^{-1}{}''u \;\&\; t \notin f^{-1}{}''v, [p]_Q \cap f^{-1}{}''u \subset' f^{-1}{}''v \vdash \\
\hline
\langle t, r\rangle \in f, r \in u, t \in [p]_Q, \mathrm{Fn}(f), r \notin v, [p]_Q \cap f^{-1}{}''u \subset' f^{-1}{}''v \vdash \\
\qquad\qquad p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash [p]_Q \cap f^{-1}{}''u \subset' f^{-1}{}''v\,; \\
\hline
\langle t, r\rangle \in f, r \in u, t \in [p]_Q, \mathrm{Fn}(f), r \notin v, p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \\
\hline
\neg\neg\exists t \in [p]_Q\,(\langle t, r\rangle \in f), r \in u, \mathrm{Fn}(f), r \notin v, p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \\
\quad r \in [q]_R, \langle p, q\rangle \in f, \mathrm{Rt}(f, Q, R) \vdash \neg\neg\exists t \in [p]_Q\,(\langle t, r\rangle \in f)\,; \\
\hline
\quad r \in [q]_R, \langle p, q\rangle \in f, \mathrm{Rt}(f, Q, R), r \in u, \mathrm{Fn}(f), r \notin v, \\
\quad p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \\
\hline
\neg\neg\, r \in [q]_R \;\&\; \neg\neg\, r \in u, \langle p, q\rangle \in f, \mathrm{Rt}(f, Q, R), r \notin v, \\
\quad p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \\
\hline
\qquad r \in [q]_R \cap u, \langle p, q\rangle \in f, \mathrm{Rt}(f, Q, R), r \notin v, \\
\qquad p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \\
\hline
\qquad\qquad \langle p, q\rangle \in f, \mathrm{Rt}(f, Q, R), \\
\qquad p \in f^{-1}{}''u \underset{Q}{\supset} f^{-1}{}''v \vdash \neg\exists r \in [q]_R \cap u\,(r \notin v)
\end{array}
\qquad (2)
$$

Схема 2.2.2

$$(2)$$

$$\langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v \vdash [q]_R \cap u \subset' v\,;$$
$$[q]_R \cap u \subset' v, q \in |R| \vdash \neg\neg\, q \in u \underset{R}{\supset} v$$

$$\langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, q \in |R| \vdash \neg\neg\, q \in u \underset{R}{\supset} v\,;$$
$$\neg\neg\, q \in u \underset{R}{\supset} v, \langle p, q \rangle \in f,$$
$$p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, q \in |R|, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\neg\neg\, q \in |R|, \langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$
$$\langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R) \vdash \neg\neg\, q \in |R|\,;$$

$$\langle p, q \rangle \in f, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\neg\neg\exists q(\langle p, q \rangle \in f), \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\neg\neg\, p \in \mathrm{Dom}(f), \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$
$$\neg\neg\, p \in |Q|, \mathrm{Rt}(f, Q, R) \vdash \neg\neg\, p \in \mathrm{Dom}(f)\,;$$

$$\neg\neg\, p \in |Q|, \mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\mathrm{Rt}(f, Q, R),$$
$$p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v, p \notin f^{-1}\text{"}(u \underset{R}{\supset} v) \vdash$$

$$\mathrm{Rt}(f, Q, R) \vdash$$
$$\vdash \neg\exists p \in f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v\,(p \notin f^{-1}\text{"}(u \underset{R}{\supset} v))$$

$$\mathrm{Rt}(f, Q, R) \vdash$$
$$\vdash f^{-1}\text{"}u \underset{Q}{\supset} f^{-1}\text{"}v \subset' f^{-1}\text{"}(u \underset{R}{\supset} v) \qquad (3)$$

Схема 2.2.3

$$\langle r, y\rangle \in f, \langle r, t\rangle \in f, \mathrm{Fn}(f) \vdash y = t\,;$$

$$y = t, t \in u \vdash y \in u$$

$$\langle r, y\rangle \in f, \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u \vdash y \in u\,;$$

$$y \in u, y \in [q]_R \vdash \neg\neg\, y \in u \cap [q]_R$$

$$\langle r, y\rangle \in f, \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u, y \in [q]_R \vdash \neg\neg\, y \in u \cap [q]_R\,;$$

$$\neg\neg\, y \in u \cap [q]_R,$$

$$u \cap [q]_R \subset' v \vdash \neg\neg\, y \in v$$

$$\langle r, y\rangle \in f, \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u, y \in [q]_R, u \cap [q]_R \subset' v \vdash \neg\neg\, y \in v\,;$$

$$\neg\neg\, y \in v, \langle r, y\rangle \in f, r \notin f^{-1}{}''v \vdash$$

$$\langle r, y\rangle \in f, \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u, y \in [q]_R, u \cap [q]_R \subset' v, r \notin f^{-1}{}''v \vdash$$

$$\neg\neg\exists y \in [q]_R\,(\langle r, y\rangle \in f), \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u, u \cap [q]_R \subset' v,$$

$$r \notin f^{-1}{}''v \vdash$$

$$r \in [p]_Q, \langle p, q\rangle \in f, \mathrm{Rt}(f,Q,R) \vdash \neg\neg\exists y \in [q]_R\,(\langle r, y\rangle \in f)\,;$$

$$r \in [p]_Q, \langle p, q\rangle \in f, \mathrm{Rt}(f,Q,R), \langle r, t\rangle \in f, \mathrm{Fn}(f), t \in u,$$

$$u \cap [q]_R \subset' v, r \notin f^{-1}{}''v \vdash$$

$$\neg\neg\exists t \in u\,(\langle r, t\rangle \in f), r \in [p]_Q, \langle p, q\rangle \in f, \mathrm{Rt}(f,Q,R),$$

$$u \cap [q]_R \subset' v, r \notin f^{-1}{}''v \vdash$$

$$r \in f^{-1}{}''u \vdash \neg\neg\exists t \in u\,(\langle r, t\rangle \in f)\,;$$

$$r \in f^{-1}{}''u, r \in [p]_Q, \langle p, q\rangle \in f, \mathrm{Rt}(f,Q,R), u \cap [q]_R \subset' v, r \notin f^{-1}{}''v \vdash$$

$$q \in u \underset{R}{\supset} v \vdash u \cap [q]_R \subset' v\,;$$

$$r \in f^{-1}{}''u, r \in [p]_Q, \langle p, q\rangle \in f, \mathrm{Rt}(f,Q,R), q \in u \underset{R}{\supset} v, r \notin f^{-1}{}''v \vdash$$

$$\neg\neg\exists q \in u \underset{R}{\supset} v\,(\langle p, q\rangle \in f), r \in f^{-1}{}''u, r \in [p]_Q, \mathrm{Rt}(f,Q,R), r \notin f^{-1}{}''v \vdash$$

$$p \in f^{-1}{}''(u \underset{R}{\supset} v) \vdash \neg\neg\exists q \in u \underset{Q}{\supset} v\,(\langle p, q\rangle \in f)\,;$$

$$p \in f^{-1}{}''(u \underset{R}{\supset} v), r \in f^{-1}{}''u, r \in [p]_Q, \mathrm{Rt}(f,Q,R), r \notin f^{-1}{}''v \vdash$$

$$\neg\neg\, r \in [p]_Q \,\&\, \neg\neg\, r \in f^{-1}{}''u, p \in f^{-1}{}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R), r \notin f^{-1}{}''v \vdash$$

$$r \in [p]_Q \cap f^{-1}{}''u, p \in f^{-1}{}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R), r \notin f^{-1}{}''v \vdash$$

$$p \in f^{-1}{}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R) \vdash$$

$$\vdash \neg\exists r \in [p]_Q \cap f^{-1}{}''u\,(r \notin f^{-1}{}''v) \qquad (4)$$

Схема 2.2.4

$$\frac{(4)}{p \in f^{-1}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R) \vdash [p]_Q \cap f^{-1}''u \subset' f^{-1}''v;}$$

$$[p]_Q \cap f^{-1}''u \subset' f^{-1}''v, \neg\neg\, p \in |Q| \vdash$$

$$\vdash \neg\neg\, p \in f^{-1}''u \underset{Q}{\supset} f^{-1}''v$$

$$\frac{}{p \in f^{-1}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R), \neg\neg\, p \in |Q| \vdash \neg\neg\, p \in f^{-1}''u \underset{Q}{\supset} f^{-1}''v}$$

$$\frac{}{p \in f^{-1}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R) \vdash \neg\neg\, p \in f^{-1}''u \underset{Q}{\supset} f^{-1}''v}$$

$$\frac{}{p \in f^{-1}''(u \underset{R}{\supset} v), \mathrm{Rt}(f,Q,R), p \notin f^{-1}''u \underset{Q}{\supset} f^{-1}''v \vdash}$$

$$\frac{}{\mathrm{Rt}(f,Q,R) \vdash}$$

$$\vdash \neg\exists p \in f^{-1}''(u \underset{R}{\supset} v)\,(p \notin f^{-1}''u \underset{Q}{\supset} f^{-1}''v)$$

$$\frac{}{\mathrm{Rt}(f,Q,R) \vdash f^{-1}''(u \underset{R}{\supset} v) \subset' f^{-1}''u \underset{Q}{\supset} f^{-1}''v} \quad (5)$$

$$\frac{q \in u, \langle q, r\rangle \in f \vdash \neg\neg\, r \in f''u}{\neg\neg\, q \in u, \langle q, r\rangle \in f, r \notin f''u \vdash}$$

$$p \in u, q \in [p]_Q, u \in \mathrm{Sec}(Q) \vdash \neg\neg\, q \in u;$$

$$\frac{}{p \in u, q \in [p]_Q, u \in \mathrm{Sec}(Q), \langle q, r\rangle \in f, r \notin f''u \vdash}$$

$$\frac{}{\neg\neg\exists q \in [p]_Q\,(\langle q, r\rangle \in f), p \in u, u \in \mathrm{Sec}(Q), r \notin f''u \vdash}$$

$$r \in [t]_R, \langle p, t\rangle \in f, \mathrm{Rt}(f,Q,R) \vdash \neg\neg\exists q \in [p]_Q\,(\langle q, r\rangle \in f);$$

$$\frac{}{r \in [t]_R, \langle p, t\rangle \in f, \mathrm{Rt}(f,Q,R), p \in u, u \in \mathrm{Sec}(Q), r \notin f''u \vdash}$$

$$\frac{}{\neg\neg\exists p \in u\,(\langle p, t\rangle \in f), r \in [t]_R, \mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q), r \notin f''u \vdash}$$

$$\frac{}{t \in f''u, r \in [t]_R, \mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q), r \notin f''u \vdash}$$

$$\frac{}{t \in f''u, r \in [t]_R, \mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q) \vdash}$$

$$\vdash \neg\neg\, r \in f''u$$

$$\frac{}{\mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q) \vdash}$$

$$\vdash \forall t \in f''u\, \forall r \in [t]_R\,(\neg\neg\, r \in f''u)$$

$$\mathrm{Rt}(f,Q,R) \vdash f''u \subset' |R|;$$

$$\frac{}{\mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q) \vdash}$$

$$\vdash f''u \subset' |R| \,\&\, \forall t \in f''u\, \forall r \in [t]_R\,(\neg\neg\, r \in f''u)$$

$$\frac{}{\mathrm{Rt}(f,Q,R), u \in \mathrm{Sec}(Q) \vdash \neg\neg\, f''u \in \mathrm{Sec}(R)} \quad (6)$$

Схема 2.2.5

$$q \in u, \langle q, r \rangle \in f^{-1} \vdash \neg\neg\, r \in f^{-1}\text{”}u$$

$$\neg\neg \langle q, r \rangle \in f^{-1}, q \in u, r \notin f^{-1}\text{”}u \vdash$$

$$\langle r, q \rangle \in f \vdash \neg\neg \langle q, r \rangle \in f^{-1};$$

$$\langle r, q \rangle \in f, q \in u, r \notin f^{-1}\text{”}u \vdash$$

$$\neg\neg\, q \in u, \langle r, q \rangle \in f, r \notin f^{-1}\text{”}u \vdash$$

$$p \in u, q \in [p]_R, u \in \mathrm{Sec}(R) \vdash \neg\neg\, q \in u;$$

$$p \in u, q \in [p]_R, u \in \mathrm{Sec}(R), \langle r, q \rangle \in f, r \notin f^{-1}\text{”}u \vdash$$

$$\neg\neg \exists q \in [p]_R (\langle r, q \rangle \in f), p \in u, u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$r \in [t]_Q, \langle t, p \rangle \in f, \mathrm{Rt}(f, Q, R) \vdash \neg\neg \exists q \in [p]_R (\langle r, q \rangle \in f);$$

$$r \in [t]_Q, \langle t, p \rangle \in f, \mathrm{Rt}(f, Q, R), p \in u, u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$\neg\neg \langle t, p \rangle \in f, r \in [t]_Q, \mathrm{Rt}(f, Q, R), p \in u, u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$\langle p, t \rangle \in f^{-1} \vdash \neg\neg \langle t, p \rangle \in f;$$

$$\langle p, t \rangle \in f^{-1}, r \in [t]_Q, \mathrm{Rt}(f, Q, R), p \in u, u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$\neg\neg \exists p \in u (\langle p, t \rangle \in f^{-1}), r \in [t]_R, \mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$t \in f^{-1}\text{”}u, r \in [t]_R, \mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R), r \notin f^{-1}\text{”}u \vdash$$

$$t \in f^{-1}\text{”}u, r \in [t]_R, \mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R) \vdash$$

$$\vdash \neg\neg\, r \in f^{-1}\text{”}u$$

$$\mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R) \vdash$$

$$\vdash \forall t \in f^{-1}\text{”}u\; \forall r \in [t]_R (\neg\neg\, r \in f^{-1}\text{”}u)$$

$$\mathrm{Rt}(f, Q, R) \vdash f^{-1}\text{”}u \subset' |Q|;$$

$$\mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R) \vdash$$

$$\vdash f^{-1}\text{”}u \subset' |Q| \;\&\; \forall t \in f^{-1}\text{”}u\; \forall r \in [t]_R (\neg\neg\, r \in f^{-1}\text{”}u)$$

$$\mathrm{Rt}(f, Q, R), u \in \mathrm{Sec}(R) \vdash \neg\neg\, f^{-1}\text{”}u \in \mathrm{Sec}(Q) \qquad (7)$$

Схема 2.2.6

$$V^C(t,S) \vdash \llbracket \check{\sigma} \in |S| \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R$$

***

$$V^C(t,S) \vdash \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in |S| \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R$$

$$V^C(S) \vdash \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in \underline{|S|}_R \rrbracket_R = \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in |S| \rrbracket_R ;$$

***

$$V^C(t,S) \vdash \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in \underline{|S|}_R \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R$$

$$V^C(S) \vdash \llbracket \check{\sigma} \in \check{\Delta} \rrbracket_R \cap \llbracket\, |S| = \check{\Delta} \rrbracket_R \subset' \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in \underline{|S|}_R \rrbracket_R ;$$

***

$$V^C(t,S) \vdash \llbracket \check{\sigma} \in \check{\Delta} \rrbracket_R \cap \llbracket\, |S| = \check{\Delta} \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R$$

***

$$V^C(t,S) \vdash \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in \check{\Delta} \rrbracket_R \cap \llbracket\, |S| = \check{\Delta} \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R$$

$$\sigma \in \Delta \vdash |R| \subset' \underset{R}{-}\underset{R}{-}\llbracket \check{\sigma} \in \check{\Delta} \rrbracket_R ;$$

***

$$\sigma \in \Delta\,, V^C(t,S) \vdash \llbracket\, |S| = \check{\Delta} \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \llbracket [\check{\sigma}]_S \cap t \neq 0 \rrbracket_R =$$

$$= \llbracket \underline{[\check{\sigma}]_S}_R \cap t \neq 0 \rrbracket_R = \llbracket \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \neq 0 \rrbracket_R = \llbracket \neg\neg\exists v (v \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t) \rrbracket_R =$$

$$= \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} \llbracket v \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \rrbracket_R = \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket v \in t \rrbracket_R)$$

***

$$\sigma \in \Delta\,, V^C(t,S) \vdash \llbracket\, |S| = \check{\Delta} \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset'$$

$$\subset' \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket v \in t \rrbracket_R) \subset'$$

$$\subset' \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket v \in t \rrbracket_R \cap \llbracket\, \underline{[\check{\sigma}]_S}_R \subset' \underline{|S|}_R \rrbracket_R \cap \Big\llbracket\, \underline{|S|}_R = \check{\Delta} \Big\rrbracket_R) \subset'$$

$$\subset' \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket v \in t \rrbracket_R \cap \llbracket \neg\neg\, v \in \check{\Delta} \rrbracket_R \cap$$

$$\cap \llbracket\, \underline{[\check{\sigma}]_S}_R \subset' \underline{|S|}_R \rrbracket_R \cap \Big\llbracket\, \underline{|S|}_R = \check{\Delta} \Big\rrbracket_R) \subset'$$

$$\subset' \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket v \in t \rrbracket_R \cap \llbracket \neg\neg\, v \in \check{\Delta} \rrbracket_R) =$$

$$= \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\underset{R}{-}\underset{R}{-}\llbracket v \in \check{\Delta} \rrbracket_R \cap \llbracket v \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \rrbracket_R) =$$

$$= \underset{R}{-}\underset{R}{-}\bigcup_{V^C(v)} (\llbracket v \in \check{\Delta} \rrbracket_R \cap \llbracket v \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \rrbracket_R) =$$

$$= \llbracket \neg\neg\exists v \in \check{\Delta} \,(v \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \,) \rrbracket_R = \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} \llbracket \check{\eta} \in \underline{[\check{\sigma}]_S}_R \,\underline{\cap}_R t \rrbracket_R =$$

$$= \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} (\llbracket \check{\eta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket \check{\eta} \in t \rrbracket_R) = \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} (\underset{R}{-}\underset{R}{-}\llbracket \check{\eta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \cap \llbracket \check{\eta} \in t \rrbracket_R) =$$

$$= \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} (\underset{R}{-}\underset{R}{-}\llbracket \check{\eta} \in [\check{\sigma}]_S \rrbracket_R \cap \llbracket \check{\eta} \in t \rrbracket_R) = \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} (\llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R \cap \llbracket \check{\eta} \in t \rrbracket_R)$$

***

$$\sigma \in \Delta\,, V^C(t,S) \vdash$$

$$\vdash \llbracket\, |S| = \check{\Delta} \rrbracket_R \cap \llbracket \mathrm{Dn}(t,S) \rrbracket_R \subset' \underset{R}{-}\underset{R}{-}\bigcup_{\eta \in \Delta} (\llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R \cap \llbracket \check{\eta} \in t \rrbracket_R) \qquad (1)$$

$$V^{C}(t,S),\, Q = R \otimes_{\Delta} S \vdash [\![\mathrm{Dn}(f^{-1}{}^{\frown}t,\, f^{-1}{}^{\frown}S)]\!]_{Q} = f^{-1}\,\text{”}\,[\![\mathrm{Dn}(t,S)]\!]_{R}$$

$$\langle p,\sigma\rangle \in [\![\mathrm{Dn}(f^{-1}{}^{\frown}t,\, f^{-1}{}^{\frown}S)]\!]_{Q},\, V^{C}(t,S),\, Q = R \otimes_{\Delta} S \vdash$$

$$\neg\neg\langle p,\sigma\rangle \in f^{-1}\,\text{”}\,[\![\mathrm{Dn}(t,S)]\!]_{R};$$

$$\neg\neg\langle p,\sigma\rangle \in f^{-1}\,\text{”}\,[\![\mathrm{Dn}(t,S)]\!]_{R},\, \neg\neg\langle\langle p,\sigma\rangle, p\rangle \in f,\, \mathrm{Fn}(f) \vdash$$

$$\vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_{R} \quad \Longleftarrow$$

$$\overset{t\quad u\quad v}{\underset{\langle p,\sigma\rangle\quad [\![\mathrm{Dn}(t,S)]\!]_{R}\quad p}{\Longleftarrow}} \quad \neg\neg\, t \in f^{-1}\,\text{”}u,\, \neg\neg\langle t, v\rangle \in f,\, \mathrm{Fn}(f) \vdash \neg\neg\, v \in u \;\; {}^{1)}$$

$$\langle p,\sigma\rangle \in [\![\mathrm{Dn}(f^{-1}{}^{\frown}t,\, f^{-1}{}^{\frown}S)]\!]_{Q},\, V^{C}(t,S),\, Q = R \otimes_{\Delta} S,$$

$$\neg\neg\langle\langle p,\sigma\rangle, p\rangle \in f,\, \mathrm{Fn}(f) \vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_{R}$$

$$\langle p,\sigma\rangle \in [\![\mathrm{Dn}(f^{-1}{}^{\frown}t,\, f^{-1}{}^{\frown}S)]\!]_{Q},\, V^{C}(t,S),\, Q = R \otimes_{\Delta} S,$$

$$p \in |R|,\, \sigma \in \Delta \vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_{R} \qquad (1)$$

$$r \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R},\, r \in [\![\check{\eta} \in t]\!]_{R},\, \eta \in \Delta \vdash \neg\neg\langle r,\eta\rangle \in \mathrm{Rf}(t,R,\Delta);$$

$$\neg\neg\langle r,\eta\rangle \in \mathrm{Rf}(t,R,\Delta),$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$

$$r \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R},\, r \in [\![\check{\eta} \in t]\!]_{R},\, \eta \in \Delta,$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\, r \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R},\, r \in [\![\check{\eta} \in t]\!]_{R},\, \eta \in \Delta,$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\, p \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R},\, r \in [p]_{R},\, V^{C}(S) \vdash$$

$$\vdash \neg\neg\, r \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R};$$

$$\neg\neg\, p \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R},\, r \in [p]_{R},\, V^{C}(S),\, r \in [\![\check{\eta} \in t]\!]_{R},\, \eta \in \Delta,$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, Q = R \otimes_{\Delta} S \vdash \neg\neg\, p \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_{R};$$

$$\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, Q = R \otimes_{\Delta} S,\, r \in [p]_{R},\, V^{C}(S),\, r \in [\![\check{\eta} \in t]\!]_{R},$$

$$\eta \in \Delta,\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\langle r,\eta\rangle \in [\langle p,\sigma\rangle]_{Q},\, Q = R \otimes_{\Delta} S,\, r \in [p]_{R},\, V^{C}(S),\, r \in [\![\check{\eta} \in t]\!]_{R},$$

$$\eta \in \Delta,\, [\langle p,\sigma\rangle]_{Q} \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash (2)$$

Схема 2.2.8

$$r \in [p]_R,\ r \in \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R,\quad \eta, \sigma \in \Delta, Q = R \otimes_\Delta S \vdash$$
$$\vdash \neg\neg \langle r, \eta \rangle \in [\langle p, \sigma \rangle]_Q; \qquad (2)$$

$$r \in [p]_R,\ r \in \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R,\quad \eta, \sigma \in \Delta, Q = R \otimes_\Delta S, r \in \llbracket \check{\eta} \in t \rrbracket_R,$$
$$V^C(S), [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$\neg\neg\, r \in \llbracket \check{\eta} \in t \rrbracket_R \,\&\, \neg\neg\, r \in \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R,\ r \in [p]_R,\quad \eta, \sigma \in \Delta,$$
$$Q = R \otimes_\Delta S, V^C(S), [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$r \in \llbracket \check{\eta} \in t \rrbracket_R \cap \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R,\ r \in [p]_R,\quad \eta, \sigma \in \Delta,$$
$$Q = R \otimes_\Delta S, V^C(S), [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$\neg\neg \exists r \in [p]_R\, \exists \eta \in \Delta\, (r \in \llbracket \check{\eta} \in t \rrbracket_R \cap \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R), \sigma \in \Delta,$$
$$Q = R \otimes_\Delta S, V^C(S), [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$p \in \llbracket \mathrm{Dn}(t, S) \rrbracket_R \cap \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta, V^C(t, S) \vdash$$
$$\vdash \neg\neg \exists r \in [p]_R\, \exists \eta \in \Delta\, (r \in \llbracket \check{\eta} \in t \rrbracket_R \cap \llbracket \neg\neg\, \check{\eta} \in [\check{\sigma}]_S \rrbracket_R)\ ^{2)};$$

$$p \in \llbracket \mathrm{Dn}(t, S) \rrbracket_R \cap \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta, V^C(t, S),$$
$$Q = R \otimes_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$\neg\neg\, p \in \llbracket \mathrm{Dn}(t, S) \rrbracket_R,\ \neg\neg\, p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta, V^C(t, S),$$
$$Q = R \otimes_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$(1) \Longrightarrow\ \langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S), Q = R \otimes_\Delta S \vdash$$
$$\vdash \neg\neg\, p \in \llbracket \mathrm{Dn}(t, S) \rrbracket_R;$$

$$\langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S), Q = R \otimes_\Delta S,$$
$$\neg\neg\, p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$\langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S), Q = R \otimes_\Delta S \vdash$$
$$\neg\neg\, p \in \llbracket \mathrm{Tv}(S) \,\&\, |S| = \check{\Delta} \rrbracket_R;$$

$$\langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S), Q = R \otimes_\Delta S,$$
$$\sigma \in \Delta, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$u = \langle p, \sigma \rangle,\ u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S), Q = R \otimes_\Delta S,$$
$$\sigma \in \Delta, [u]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash$$

$$\neg\neg \exists p\, \exists \sigma \in \Delta\, (u = \langle p, \sigma \rangle),\ u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t, S),$$
$$Q = R \otimes_\Delta S, [u]_Q \cap \mathrm{Rf}(t, R, \Delta) = 0 \vdash \qquad (3)$$

$$u \in [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q,\ V^C(t,S),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash \neg\neg \exists p\ \exists \sigma \in \Delta\, (u = \langle p, \sigma \rangle); \quad (3)$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$u \in [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q,\ V^C(t,S),\ Q = R \otimes_\Delta S,$$
$$[u]_Q \cap \mathrm{Rf}(t,R,\Delta) = 0 \vdash$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$V^C(t,S),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash \forall u \in [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q\, ([u]_Q \cap \mathrm{Rf}(t,R,\Delta) \neq 0)$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$V^C(t,S),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash \mathrm{Dn}(\mathrm{Rf}(t,R,\Delta), [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q, Q);$$
$$\mathrm{Dn}(\mathrm{Rf}(t,R,\Delta), [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q, Q),\ V^C(t,S),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_R$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$V^C(t,S),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_R \qquad (4)$$

$$\neg\neg\, q \in [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ \eta \in \Delta \vdash \neg\neg \langle\langle q, \eta \rangle, q \rangle \in f;$$
$$\neg\neg \langle\langle q, \eta \rangle, q \rangle \in f,\ q \in [\![\check{\sigma} \in t]\!]_R \vdash \neg\neg \langle q, \eta \rangle \in f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$\neg\neg\, q \in [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ \eta \in \Delta,\ q \in [\![\check{\sigma} \in t]\!]_R \vdash$$
$$\vdash \neg\neg \langle q, \eta \rangle \in f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R;$$
$$V^C(t),\ Q = R \otimes_\Delta S \vdash f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R \subset' [\![f^{-1}{}^\frown \check{\sigma} \in f^{-1}{}^\frown t]\!]_Q =$$
$$= [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$\neg\neg\, q \in [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ \eta \in \Delta,\ q \in [\![\check{\sigma} \in t]\!]_R,\ V^C(t),\ Q = R \otimes_\Delta S \vdash$$
$$\vdash \neg\neg \langle q, \eta \rangle \in [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q;$$
$$\neg\neg \langle q, \eta \rangle \in [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q,$$
$$\langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg \langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$\neg\neg\, q \in [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ \eta \in \Delta,\ q \in [\![\check{\sigma} \in t]\!]_R,\ V^C(t),\ Q = R \otimes_\Delta S,$$
$$\langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg \langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q$$
$$\langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q,\ Q = R \otimes_\Delta S \vdash \neg\neg\, q \in [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R;$$
$$\underline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$
$$\langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ q \in [\![\check{\sigma} \in t]\!]_R,\ V^C(t) \vdash$$
$$\vdash \neg\neg \langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \ (5)$$

$$\frac{(5)}{\neg\neg\, q \in [\![\check{\sigma} \in t]\!]_R,\ \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1} {}^\frown t]\!]_Q}$$

$$\frac{q \in [r]_R,\ r \in [\![\check{\sigma} \in t]\!]_R \vdash \neg\neg\, q \in [\![\check{\sigma} \in t]\!]_R;}{q \in [r]_R,\ r \in [\![\check{\sigma} \in t]\!]_R,\ \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1} {}^\frown t]\!]_Q}$$

$$\frac{\neg\neg\, q \in [r]_R,\ r \in [\![\check{\sigma} \in t]\!]_R,\ \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1} {}^\frown t]\!]_Q \qquad \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S, \vdash \neg\neg\, q \in [r]_R;}{r \in [\![\check{\sigma} \in t]\!]_R,\ \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1} {}^\frown t]\!]_Q \subset' \subset' \bigcup_{p \in [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R} \bigcup_{\zeta \in \Delta} ([\langle p,\zeta\rangle]_Q \cap [\![\check{\zeta} \in f^{-1} {}^\frown t]\!]_Q) \subset' \subset' [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q}$$

$$\frac{r \in [\![\check{\sigma} \in t]\!]_R,\ \langle q,\eta\rangle \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg \langle q,\eta\rangle \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q}{p = \langle q,\eta\rangle, p \in [\langle r,\sigma\rangle]_Q,\ r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg\, p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q}$$

$$\frac{\neg\neg \exists q\, \exists \eta \in \Delta\, (p = \langle q,\eta\rangle),\ p \in [\langle r,\sigma\rangle]_Q\ r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg\, p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q \qquad p \in [\langle r,\sigma\rangle]_Q,\ Q = R \otimes_\Delta S \vdash \neg\neg \exists q\, \exists \eta \in \Delta\, (p = \langle q,\eta\rangle);}{p \in [\langle r,\sigma\rangle]_Q,\ r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t) \vdash \neg\neg\, p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q}$$

$$\frac{p \in [\langle r,\sigma\rangle]_Q,\ r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t),\ p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q \vdash}{\neg\neg\, p \in [\langle r,\sigma\rangle]_Q, r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \eta \in \Delta,\ V^C(t),\ p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q \vdash}$$

$$\neg\neg\, p \in [\langle r,\sigma\rangle]_Q, r \in [\![\check{\sigma} \in t]\!]_R,\ Q = R \otimes_\Delta S,\ \neg\neg \exists \eta (\eta \in \Delta),\ V^C(t),\ p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1} {}^\frown t \neq 0]\!]_Q \vdash\ (6)$$

$$p \in [q]_Q\,, q = \langle r, \sigma \rangle \vdash \neg\neg\, p \in [\langle r, \sigma \rangle]_Q\,;$$

$$(6) \Longrightarrow\ \neg\neg\, p \in [\langle r, \sigma \rangle]_Q\,, r \in [\![\check{\sigma} \in t]\!]_R\,, Q = R \otimes_\Delta S,$$
$$\Delta \neq 0\,, V^C(t), p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash$$

$$p \in [q]_Q\,, q = \langle r, \sigma \rangle, r \in [\![\check{\sigma} \in t]\!]_R\,, Q = R \otimes_\Delta S,$$
$$\Delta \neq 0\,, V^C(t), p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q$$

$$\neg\neg\exists r \in [\![\mathrm{Tv}(R)\ \&\ |S| = \check{\Delta}]\!]_R\ \exists \sigma \in \Delta\, (r \in [\![\check{\sigma} \in t]\!]_R\ \&\ q = \langle r, \sigma \rangle),$$
$$p \in [q]_Q\,, Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t), p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash$$
$$q \in \mathrm{Rf}(t, R, \Delta) \vdash \neg\neg\exists \sigma \in \Delta\ \exists r \in [\![\check{\sigma} \in t]\!]_R\, (q = \langle r, \sigma \rangle)\,;$$

$$q \in \mathrm{Rf}(t, R, \Delta), p \in [q]_Q\,, Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t),$$
$$p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash$$

$$\neg\neg\exists q \in \mathrm{Rf}(t, R, \Delta)\, (p \in [q]_Q), Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t),$$
$$p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash$$

$$p \in \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_Q\,, Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t),$$
$$p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash$$

$$Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t) \vdash$$
$$\vdash \neg\exists p \in \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_Q\, (p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q)$$

$$Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t) \vdash$$
$$\vdash \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_Q \subset' [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q$$

$$Q = R \otimes_\Delta S, \Delta \neq 0\,, V^C(t) \vdash$$
$$\vdash \overline{}_Q\, \overline{}_Q \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_Q \subset' [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q$$

$$(4) \Longrightarrow\ V^C(t, S), Q = R \otimes_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t, f^{-1}{}^\frown S)]\!]_Q \subset' \overline{}_Q\, \overline{}_Q \bigcup_{q \in \mathrm{Rf}(t,R,\Delta)} [q]_R\,;$$

$$V^C(t, S), Q = R \otimes_\Delta S, \Delta \neq 0 \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t, f^{-1}{}^\frown S)]\!]_Q \subset' [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \qquad (7)$$

$$V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, Q = R \otimes'_\Delta S \vdash$$
$$\vdash [\![\mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S)]\!]_Q = f^{-1}\text{''}\, [\![\mathrm{Dn}(t,S)]\!]_R$$

$$\langle p, \sigma \rangle \in [\![\mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S)]\!]_Q, V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R,$$
$$Q = R \otimes'_\Delta S \vdash$$
$$\vdash \neg\neg \langle p, \sigma \rangle \in f^{-1}\text{''}\, [\![\mathrm{Dn}(t,S)]\!]_R ;$$
$$\neg\neg \langle p, \sigma \rangle \in f^{-1}\text{''}\, [\![\mathrm{Dn}(t,S)]\!]_R, \neg\neg \langle \langle p, \sigma \rangle, p \rangle \in f, \mathrm{Fn}(f) \vdash$$
$$\vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_R \quad \Longleftarrow$$
$$\underset{\langle p,\sigma\rangle}{\overset{t}{}}\ \underset{[\![\mathrm{Dn}(t,S)]\!]_R}{\overset{u}{\Longleftarrow}}\ \underset{p}{\overset{v}{}} \qquad \neg\neg\, t \in f^{-1}\text{''} u, \neg\neg \langle t, v \rangle \in f, \mathrm{Fn}(f) \vdash \neg\neg\, v \in u$$ [1)]

$$\langle p, \sigma \rangle \in [\![\mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S)]\!]_Q, V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R,$$
$$Q = R \otimes'_\Delta S, \neg\neg \langle \langle p, \sigma \rangle, p \rangle \in f, \mathrm{Fn}(f) \vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_R$$

$$\langle p, \sigma \rangle \in [\![\mathrm{Dn}(f^{-1} {}^\frown t, f^{-1} {}^\frown S)]\!]_Q, V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R,$$
$$Q = R \otimes'_\Delta S, p \in |R|, \sigma \in \Delta \vdash \neg\neg\, p \in [\![\mathrm{Dn}(t,S)]\!]_R \qquad (1)$$

$$r \in [\![\check{\eta} \in t]\!]_R, \eta \in \Delta \vdash \neg\neg \langle r, \eta \rangle \in \mathrm{Rf}'(t,R,\Delta);$$
$$\neg\neg \langle r, \eta \rangle \in \mathrm{Rf}'(t,R,\Delta),$$
$$\langle r, \eta \rangle \in [\langle p, \sigma \rangle]_Q, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$r \in [\![\check{\eta} \in t]\!]_R, \eta \in \Delta, \langle r, \eta \rangle \in [\langle p, \sigma \rangle]_Q, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg \langle r, \eta \rangle \in [\langle p, \sigma \rangle]_Q, r \in [\![\check{\eta} \in t]\!]_R, \eta \in \Delta, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$
$$r \in [p]_R, r \in [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R, \quad \eta, \sigma \in \Delta, Q = R \otimes'_\Delta S \vdash$$
$$\vdash \neg\neg \langle r, \eta \rangle \in [\langle p, \sigma \rangle]_Q ;$$

$$r \in [p]_R, r \in [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R, \quad \eta, \sigma \in \Delta, Q = R \otimes'_\Delta S, r \in [\![\check{\eta} \in t]\!]_R,$$
$$[\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\, r \in [\![\check{\eta} \in t]\!]_R \,\&\, \neg\neg\, r \in [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R, r \in [p]_R, \quad \eta, \sigma \in \Delta,$$
$$Q = R \otimes'_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$r \in [\![\check{\eta} \in t]\!]_R \cap [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R, r \in [p]_R, \quad \eta, \sigma \in \Delta,$$
$$Q = R \otimes'_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg \exists r \in [p]_R\, \exists \eta \in \Delta\, (r \in [\![\check{\eta} \in t]\!]_R \cap [\![\neg\neg\, \check{\eta} \in [\check{\sigma}]_S]\!]_R), \sigma \in \Delta,$$
$$Q = R \otimes'_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash \qquad (2)$$

$$p \in \llbracket \mathrm{Dn}(t,S) \rrbracket_R,\ V^C(t,S),\ |R| \subset' \llbracket\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta, \vdash$$
$$\vdash \neg\neg \exists r \in [p]_R\ \exists \eta \in \Delta\ (r \in \llbracket \check{\eta} \in t \rrbracket_R \cap \llbracket \neg\neg\ \check{\eta} \in [\check{\sigma}]_S \rrbracket_R)\ {}^{2)};\quad (2)$$

$$p \in \llbracket \mathrm{Dn}(t,S) \rrbracket_R,\ V^C(t,S),\ |R| \subset' \llbracket\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta,$$
$$Q = R \otimes_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\ p \in \llbracket \mathrm{Dn}(t,S) \rrbracket_R,\ V^C(t,S),\ |R| \subset' \llbracket\, |S| = \check{\Delta} \rrbracket_R,\ \sigma \in \Delta,$$
$$Q = R \otimes'_\Delta S, [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash \qquad (1) \Rightarrow$$
$$\Rightarrow \langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),\ |R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,$$
$$Q = R \otimes'_\Delta S,\ p \in |R|,\ \sigma \in \Delta \vdash \neg\neg\ p \in \llbracket \mathrm{Dn}(t,S) \rrbracket_R;$$

$$\langle p, \sigma \rangle \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),\ |R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,$$
$$Q = R \otimes'_\Delta S, p \in |R|,\ \sigma \in \Delta,\ [\langle p, \sigma \rangle]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$u = \langle p, \sigma \rangle,\ u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),\ p \in |R|,\ \sigma \in \Delta,$$
$$|R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S,\ [u]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg \exists p \in |R|\ \exists \sigma \in \Delta\ (u = \langle p, \sigma \rangle),\ u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),$$
$$|R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S,\ [u]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$\neg\neg\ u \in |R| \times \Delta,\ u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),$$
$$|R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S,\ [u]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q,\ V^C(t,S),$$
$$|R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S,\ [u]_Q \cap \mathrm{Rf}'(t,R,\Delta) = 0 \vdash$$

$$V^C(t,S),\ |R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S \vdash$$
$$\vdash \forall u \in \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q\ ([u]_Q \cap \mathrm{Rf}'(t,R,\Delta) \neq 0)$$

$$V^C(t,S),\ |R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S \vdash$$
$$\vdash \mathrm{Dn}(\mathrm{Rf}'(t,R,\Delta), \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q, Q);$$
$$\mathrm{Dn}(\mathrm{Rf}'(t,R,\Delta), \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q, Q),\ V^C(t,S),$$
$$|R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S \vdash$$
$$\vdash \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_R$$

$$V^C(t,S),\ |R| \subset' \llbracket \mathrm{Tv}(S)\ \&\ |S| = \check{\Delta} \rrbracket_R,\ Q = R \otimes'_\Delta S \vdash$$
$$\vdash \llbracket \mathrm{Dn}(f^{-1} {}^\frown t,\ f^{-1} {}^\frown S) \rrbracket_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_R \qquad (3)$$

$$\neg\neg\, q \in |R|, \eta \in \Delta \vdash \neg\neg \langle\langle q, \eta\rangle, q\rangle \in f\,;$$

$$\frac{\neg\neg \langle\langle q, \eta\rangle, q\rangle \in f, q \in [\![\check{\sigma} \in t]\!]_R \vdash \neg\neg \langle q, \eta\rangle \in f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R}{\dfrac{\neg\neg\, q \in |R|, \eta \in \Delta, q \in [\![\check{\sigma} \in t]\!]_R \vdash \neg\neg \langle q, \eta\rangle \in f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R}{\eta \in \Delta, q \in [\![\check{\sigma} \in t]\!]_R \vdash \neg\neg \langle q, \eta\rangle \in f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R\,;}}$$

$$\frac{\begin{array}{c} V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R \vdash \\ \vdash f^{-1}\text{''}[\![\check{\sigma} \in t]\!]_R \subset' [\![f^{-1}{}^\frown\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q = [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \end{array}}{\begin{array}{c} \eta \in \Delta, q \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R \vdash \\ \vdash \neg\neg \langle q, \eta\rangle \in [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q\,; \end{array}}$$

$$\frac{\begin{array}{c} \neg\neg \langle q, \eta\rangle \in [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q, \\ \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \end{array}}{\dfrac{\begin{array}{c} \eta \in \Delta, q \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \end{array}}{\begin{array}{c} \neg\neg\, q \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \eta \in \Delta, \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \end{array}}}$$

$$\frac{q \in [r]_R, r \in [\![\check{\sigma} \in t]\!]_R, V^C(t) \vdash \neg\neg\, q \in [\![\check{\sigma} \in t]\!]_R\,;}{\begin{array}{c} q \in [r]_R, r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \eta \in \Delta, \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \end{array}}$$

$$\frac{\begin{array}{c} \neg\neg\, q \in [r]_R, r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \\ |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \eta \in \Delta, \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \\ \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg\, q \in [r]_R\,; \end{array}}{\begin{array}{c} r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \eta \in \Delta, \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \cap [\![\check{\sigma} \in f^{-1}{}^\frown t]\!]_Q \subset' \\ \subset' \bigcup_{p \in |R|} \bigcup_{\zeta \in \Delta} ([\langle p, \zeta\rangle]_Q \cap [\![\check{\zeta} \in f^{-1}{}^\frown t]\!]_Q) \subset' \\ \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \end{array}}$$

$$\begin{array}{c} r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\,\&\,|S| = \check{\Delta}]\!]_R, \\ \eta \in \Delta, \langle q, \eta\rangle \in [\langle r, \sigma\rangle]_Q \vdash \neg\neg \langle q, \eta\rangle \in [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \end{array} \quad (4)$$

$$\frac{\dfrac{\dfrac{(4) \Longrightarrow\ r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad \eta \in \Delta, \langle q, \eta \rangle \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg \langle q, \eta \rangle \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q}{p = \langle q, \eta \rangle, r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad \eta \in \Delta, p \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q}}{\neg\neg\exists q\, \exists \eta \in \Delta\, (p = \langle q, \eta \rangle), r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad p \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \qquad p \in [\langle r, \sigma \rangle]_Q, Q = R \otimes'_\Delta S \vdash \neg\neg\exists q\, \exists \eta \in \Delta\, (p = \langle q, \eta \rangle);}}{r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad p \in [\langle r, \sigma \rangle]_Q \vdash \neg\neg p \in [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q}$$

$$\frac{r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad p \in [\langle r, \sigma \rangle]_Q, p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash}{r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad \neg\neg p \in [\langle r, \sigma \rangle]_Q, p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash \qquad p \in [q]_Q, q = \langle r, \sigma \rangle \vdash \neg\neg p \in [\langle r, \sigma \rangle]_Q;}$$

$$\frac{\dfrac{p \in [q]_Q, q = \langle r, \sigma \rangle, r \in [\![\check{\sigma} \in t]\!]_R, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash}{\neg\neg\exists \sigma \in \Delta\, \exists r \in [\![\check{\sigma} \in t]\!]_R\, (q = \langle r, \sigma \rangle), p \in [q]_Q, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, \quad p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash \qquad q \in \mathrm{Rf}'(t, R, \Delta) \vdash \neg\neg\exists \sigma \in \Delta\, \exists r \in [\![\check{\sigma} \in t]\!]_R\, (q = \langle r, \sigma \rangle);}}{q \in \mathrm{Rf}'(t, R, \Delta), p \in [q]_Q, V^C(t), Q = R \otimes'_\Delta S, \quad |R| \subset' [\![\mathrm{Tv}(S) \,\&\, |S| = \check{\Delta}]\!]_R, p \notin [\![\underline{\mathrm{Rg}_Q}(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash \qquad (5)}$$

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$$(5) \Longrightarrow q \in \mathrm{Rf}'(t,R,\Delta),\ p \in [q]_Q,\ V^C(t),\ Q = R \otimes'_\Delta S,$$
$$\dfrac{|R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ p \notin [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash}{\neg\neg\exists q \in \mathrm{Rf}'(t,R,\Delta)\,(p \in [q]_Q),\ V^C(t),\ Q = R \otimes'_\Delta S,}$$
$$\dfrac{|R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ p \notin [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash}{p \in \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_Q,\ V^C(t),\ Q = R \otimes'_\Delta S,}$$
$$\dfrac{|R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ p \notin [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \vdash}{V^C(t),\ Q = R \otimes'_\Delta S,\quad |R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R \vdash}$$
$$\dfrac{\neg\exists p \in \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_Q\,(p \notin [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q)}{V^C(t),\ Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R \vdash}$$
$$\dfrac{\vdash \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q}{V^C(t),\ Q = R \otimes'_\Delta S, |R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R \vdash}$$
$$\vdash_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q$$
$$(3) \Longrightarrow V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ Q = R \otimes'_\Delta S \vdash$$
$$\dfrac{\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q \subset'_{\overline{Q}\,\overline{Q}} \bigcup_{q \in \mathrm{Rf}'(t,R,\Delta)} [q]_R;}{V^C(t,S), |R| \subset' [\![\mathrm{Tv}(S)\ \&\ |S| = \check{\Delta}]\!]_R,\ Q = R \otimes'_\Delta S \vdash}$$
$$\vdash [\![\mathrm{Dn}(f^{-1}{}^\frown t,\ f^{-1}{}^\frown S)]\!]_Q \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap f^{-1}{}^\frown t \neq 0]\!]_Q \qquad (6)$$

$$\begin{aligned}
\vdash [\![t \in \mathring{S}]\!]_R &= \bigcup_{r \in \mathrm{Dom}(\mathring{S})} (\mathring{S}(r) \cap [\![t = r]\!]_R) = \\
&= \bigcup_{r \in \mathrm{Dom}(\mathring{S})} (\bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} v(r)) \cap [\![t = r]\!]_R) = \\
&= \bigcup_{r \in \mathrm{Dom}(\mathring{S})} \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} v(r) \cap [\![t = r]\!]_R) = \\
&= \bigcup_{r \in \mathrm{Dom}(\mathring{S})} \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} (v(r) \cap [\![t = r]\!]_R)) = \\
&= \bigcup_{u \in \mathrm{Dom}(S)} \bigcup_{r \in \mathrm{Dom}(\mathring{S})} (u \cap \bigcup_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} (v(r) \cap [\![t = r]\!]_R)) = \\
&= \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{r \in \mathrm{Dom}(\mathring{S})} \bigcup_{\substack{v \in S[u],\\ r \in \mathrm{Dom}(v)}} (v(r) \cap [\![t = r]\!]_R)) = \\
&= \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{v \in S[u]} \bigcup_{r \in \mathrm{Dom}(v)} (v(r) \cap [\![t = r]\!]_R)) = \\
&= \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{v \in S[u]} [\![t \in v]\!]_R)
\end{aligned}$$

$$\overline{\vdash [\![t \in \mathring{S}]\!]_R = \bigcup_{u \in \mathrm{Dom}(S)} (u \cap \bigcup_{v \in S[u]} [\![t \in v]\!]_R)} \qquad (1)$$

$$\underline{u \in Z, \langle u, v \rangle \in S \vdash \neg\neg v \in S\,"Z}$$

$$\underline{u \in Z, \langle u, v \rangle \in S, v \notin S\,"Z \vdash}$$

$$\underline{\neg\neg \langle u, v \rangle \in S, u \in Z, v \notin S\,"Z \vdash}$$

$$\underline{v \in S[u], u \in Z, v \notin S\,"Z \vdash}$$

$$\underline{u \in Z, v \in S[u], v \notin S\,"Z \vdash}$$

$$v \in S[u], v \notin S\,"Z \vdash u \notin Z\,;$$

$$\underline{u \notin Z, u \in \mathrm{Dom}(S) \vdash \neg\neg u \in \mathrm{Dom}(S) - Z}$$

$$v \in S[u], v \notin S\,"Z, u \in \mathrm{Dom}(S) \vdash \neg\neg u \in \mathrm{Dom}(S) - Z\,;$$

$$\underline{\neg\neg u \in \mathrm{Dom}(S) - Z, r \in u \vdash \neg\neg r \in \bigcup(\mathrm{Dom}(S) - Z)}$$

$$\underline{v \in S[u], v \notin S\,"Z, u \in \mathrm{Dom}(S), r \in u \vdash \neg\neg r \in \bigcup(\mathrm{Dom}(S) - Z)}$$

$$\underline{v \in S[u], v \notin S\,"Z, u \in \mathrm{Dom}(S), r \in u, r \notin \bigcup(\mathrm{Dom}(S) - Z) \vdash}$$

$$\underline{v \in S[u], v \notin S\,"Z, u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S} \vdash}$$

$$\underline{v \notin S\,"Z, v \in S[u], u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S} \vdash}$$

$$v \in S[u], u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S} \vdash \neg\neg v \in S\,"Z \qquad (2)$$

$$(2) \Longrightarrow \; v \in S[u]\,, u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S} \vdash \neg\neg\, v \in S\,{}''Z\,;$$

$$\neg\neg\, v \in S\,{}''Z, r \in \llbracket t \in v \rrbracket_R \vdash \neg\neg\, r \in \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R$$

$$v \in S[u]\,, u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S}, r \in \llbracket t \in v \rrbracket_R \vdash \neg\neg\, r \in \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R$$

$$v \in S[u]\,, u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S}, r \in \llbracket t \in v \rrbracket_R\,, r \notin \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R \vdash$$

$$\neg\neg\exists v \in S[u]\,(r \in \llbracket t \in v \rrbracket_R), u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$r \in \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,, u \in \mathrm{Dom}(S), r \in u, r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$\neg\neg\, r \in u \,\&\, \neg\neg\, r \in \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,, u \in \mathrm{Dom}(S), r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$r \in u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,, u \in \mathrm{Dom}(S), r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$\neg\neg\exists u \in \mathrm{Dom}(S)\,(\, r \in u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,), r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$r \in \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,), r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$\neg\neg\, r \in \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,), r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$\llbracket t \in \mathring{S} \rrbracket_R = \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,), r \in \llbracket t \in \mathring{S} \rrbracket_R \vdash$$

$$\vdash \neg\neg\, r \in \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,)\,;$$

$$\llbracket t \in \mathring{S} \rrbracket_R = \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,), r \in \llbracket t \in \mathring{S} \rrbracket_R\,,$$

$$r \in Z_{\frown S}, r \notin \ldots \vdash$$

$$(1) \Longrightarrow \vdash \llbracket t \in \mathring{S} \rrbracket_R = \bigcup_{u \in \mathrm{Dom}(S)} (\, u \cap \bigcup_{v \in S[u]} \llbracket t \in v \rrbracket_R\,)\,;$$

$$r \in \llbracket t \in \mathring{S} \rrbracket_R\,, r \in Z_{\frown S}\,, r \notin \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R \vdash$$

$$\neg\neg\, r \in Z_{\frown S} \,\&\, \neg\neg\, r \in \llbracket t \in \mathring{S} \rrbracket_R\,, r \notin \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R \vdash$$

$$r \in Z_{\frown S} \cap \llbracket t \in \mathring{S} \rrbracket_R, r \notin \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R \vdash$$

$$\vdash \neg\exists r \in Z_{\frown S} \cap \llbracket t \in \mathring{S} \rrbracket_R\,(\, r \notin \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R\,)$$

$$\vdash Z_{\frown S} \cap \llbracket t \in \mathring{S} \rrbracket_R \subset' \bigcup_{y \in S\,{}''Z} \llbracket t \in y \rrbracket_R \qquad (3)$$

Схема 2.3.3

$$
\begin{array}{l}
\vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R = \\
= \bigcap Z \cap \bigcup\limits_{u \in Z} \; \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R = \\
= \bigcup\limits_{u \in Z} \Big(\bigcap Z \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \\
\hline
\vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R = \bigcup\limits_{u \in Z} \Big(\bigcap Z \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) ; \\
\forall u \in Z \, (\bigcap Z \subset' u) \vdash \\
\vdash \bigcup\limits_{u \in Z} \Big(\bigcap Z \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \subset' \bigcup\limits_{u \in Z} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \\
\hline
\forall u \in Z \, (\bigcap Z \subset' u) \vdash \\
\vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \subset' \bigcup\limits_{u \in Z} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \\
\vdash \forall u \in Z \, (\bigcap Z \subset' u) ; \\
\hline
\vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \subset' \bigcup\limits_{u \in Z} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) ; \\
Z \subset' \mathrm{Dom}(S) \vdash \\
\vdash \bigcup\limits_{u \in Z} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \subset' \bigcup\limits_{u \in \mathrm{Dom}(S)} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \\
\hline
Z \subset' \mathrm{Dom}(S) \vdash \\
\qquad \vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \subset' \bigcup\limits_{u \in \mathrm{Dom}(S)} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) \\
\qquad\qquad (1) \Longrightarrow \; \vdash \llbracket t \in \mathring{S} \rrbracket_R = \bigcup\limits_{u \in \mathrm{Dom}(S)} \Big(u \cap \bigcup\limits_{v \in S[u]} \llbracket t \in v \rrbracket_R \Big) ; \\
\hline
Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \subset' \llbracket t \in \mathring{S} \rrbracket_R
\end{array}
\qquad (4)
$$

$$
\begin{array}{l}
\qquad\qquad (4) ; \qquad\qquad \vdash \llbracket t \in \mathring{S} \rrbracket_R \cap -_R \llbracket t \in \mathring{S} \rrbracket_R \subset' 0 \\
\hline
Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \cap -_R \llbracket t \in \mathring{S} \rrbracket_R \subset' 0 ; \\
\qquad\qquad\qquad \bigcap Z \cap \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \cap -_R \llbracket t \in \mathring{S} \rrbracket_R \subset' 0 , \\
\neg\neg \bigcap Z , \; -_R \llbracket t \in \mathring{S} \rrbracket_R \in \mathrm{O}(R) \vdash \bigcap Z \cap -_R \llbracket t \in \mathring{S} \rrbracket_R \subset' -_R \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R \\
\hline
\neg\neg \bigcap Z , \; -_R \llbracket t \in \mathring{S} \rrbracket_R \in \mathrm{O}(R) , \; Z \subset' \mathrm{Dom}(S) \vdash \\
\quad \vdash \bigcap Z \cap -_R \llbracket t \in \mathring{S} \rrbracket_R \subset' -_R \bigcup\limits_{v \in S\,"Z} \llbracket t \in v \rrbracket_R
\end{array}
\qquad (5)
$$

Схема 2.3.4

$$\vdash \neg\neg \underset{R}{-}[\![t \in \mathring{S}]\!]_R \in \mathrm{O}(R)\,;$$

$$\neg\neg \bigcap Z,\ \underset{R}{-}[\![t \in \mathring{S}]\!]_R \in \mathrm{O}(R),\ Z \subset' \mathrm{Dom}(S) \vdash$$

$$\vdash \bigcap Z \cap \underset{R}{-}[\![t \in \mathring{S}]\!]_R \subset' \underset{R}{-} \bigcup_{v \in S''Z} [\![t \in v]\!]_R \ \Longleftarrow (5)$$

$$\dfrac{}{\neg\neg \bigcap Z \in \mathrm{O}(R),\ Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \underset{R}{-}[\![t \in \mathring{S}]\!]_R \subset' \underset{R}{-} \bigcup_{v \in S''Z} [\![t \in v]\!]_R}$$

$$\dfrac{}{\neg\neg \bigcap Z \in \mathrm{O}(R),\ Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \underset{R}{-}\underset{R}{-} \bigcup_{v \in S''Z} [\![t \in v]\!]_R \subset' \underset{R}{-}\underset{R}{-}[\![t \in \mathring{S}]\!]_R}$$

$$Z \subset' \mathrm{O}(R) \vdash \neg\neg \bigcap Z \in \mathrm{O}(R)\,;$$

$$\dfrac{}{Z \subset' \mathrm{O}(R),\ Z \subset' \mathrm{Dom}(S) \vdash \bigcap Z \cap \underset{R}{-}\underset{R}{-} \bigcup_{v \in S''Z} [\![t \in v]\!]_R \subset' \underset{R}{-}\underset{R}{-}[\![t \in \mathring{S}]\!]_R \ (6)}$$

$$\dfrac{p \in \mathrm{Dom}(u,r,R) \vdash \neg\neg\, r \in [p]_R\,;\quad \neg\neg\, r \in [p]_R,\ r \notin v,\ v = [p]_R \vdash}{p \in \mathrm{Dom}(u,r,R),\ r \notin v,\ v = [p]_R \vdash}$$

$$\dfrac{}{\neg\neg \exists p \in \mathrm{Dom}(u,r,R)\ (v = [p]_R),\ r \notin v \vdash}$$

$$\dfrac{}{v \in [\mathrm{Dom}(u,r,R)]^R,\ r \notin v \vdash}$$

$$\dfrac{}{v \in [\mathrm{Dom}(u,r,R)]^R \vdash \neg\neg\, r \in v}$$

$$\dfrac{}{\vdash \forall v \in [\mathrm{Dom}(u,r,R)]^R\ \neg\neg\, r \in v\,;}$$

$$\forall v \in [\mathrm{Dom}(u,r,R)]^R\ \neg\neg\, r \in v,$$

$$[\mathrm{Dom}(u,r,R)]^R \neq 0 \vdash \neg\neg\, r \in \bigcap\ [\mathrm{Dom}(u,r,R)]^R$$

$$\dfrac{}{[\mathrm{Dom}(u,r,R)]^R \neq 0 \vdash \neg\neg\, r \in \bigcap\ [\mathrm{Dom}(u,r,R)]^R}$$

$$\dfrac{\mathrm{Dom}(u,r,R) \neq 0 \vdash [\mathrm{Dom}(u,r,R)]^R \neq 0\,;}{\mathrm{Dom}(u,r,R) \neq 0 \vdash \neg\neg\, r \in \bigcap\ [\mathrm{Dom}(u,r,R)]^R \qquad (7)}$$

$$r \in [p]_R,\ p \in \mathrm{Dom}(u) \vdash \neg\neg\, p \in \mathrm{Dom}(u,r,R)\,;$$

$$\dfrac{\neg\neg\, p \in \mathrm{Dom}(u,r,R),\ \mathrm{Dom}(u,r,R) = 0 \vdash}{r \in [p]_R,\ p \in \mathrm{Dom}(u),\ \mathrm{Dom}(u,r,R) = 0 \vdash}$$

$$\dfrac{}{\neg\neg \exists p \in \mathrm{Dom}(u)\ (r \in [p]_R),\ \mathrm{Dom}(u,r,R) = 0 \vdash}$$

$$\dfrac{}{r \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R,\ \mathrm{Dom}(u,r,R) = 0 \vdash}$$

$$\dfrac{}{r \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \vdash \mathrm{Dom}(u,r,R) \neq 0\,; \qquad (7)}$$

$$\dfrac{}{r \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \vdash \neg\neg\, r \in \bigcap\ [\mathrm{Dom}(u,r,R)]^R \qquad (8)}$$

$$\cfrac{\cfrac{r\in[p]_R,\ p\in \mathrm{Dom}(u)\vdash \neg\neg\, p\in \mathrm{Dom}(u,r,R)\,;\qquad \neg\neg\, p\in \mathrm{Dom}(u,r,R),\ t=[p]_R\vdash\ \vdash \neg\neg\, t\in[\mathrm{Dom}(u,r,R)]^R}{r\in[p]_R,\ p\in \mathrm{Dom}(u),\ t=[p]_R\vdash \neg\neg\, t\in[\mathrm{Dom}(u,r,R)]^R}}{r\in[p]_R,\ p\in \mathrm{Dom}(u),\ t=[p]_R,\ t\notin[\mathrm{Dom}(u,r,R)]^R\vdash}$$

$$\cfrac{p\in \mathrm{Dom}(u),\ t=[p]_R,\ t\notin[\mathrm{Dom}(u,r,R)]^R\vdash r\notin[p]_R\,;\qquad r\notin[p]_R,\ r\in t,\ t=[p]_R\vdash}{p\in \mathrm{Dom}(u),\ t=[p]_R,\ t\notin[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash}$$

$$\cfrac{\neg\neg\exists p\in \mathrm{Dom}(u)\,(t=[p]_R),\ t\notin[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash\qquad t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S))\vdash \neg\neg\exists p\in \mathrm{Dom}(u)\,(t=[p]_R)\,;}{t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S)),\ t\notin[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash}$$

$$\cfrac{\neg\neg\, t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S)),\ t\notin[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash}{\cfrac{\neg\neg\, t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S))\ \&\ t\notin[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash}{\cfrac{t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R,\ r\in t\vdash}{\cfrac{\neg\neg\exists t\in \mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R\,(r\in t)\vdash}{\cfrac{r\in\bigcup(\mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R)\vdash}{\vdash r\notin\bigcup(\mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R)}}}}} \qquad (9)$$

$$\cfrac{(8)\Longrightarrow\quad r\in\bigcup_{p\in \mathrm{Dom}(u)}[p]_R\vdash \neg\neg\, r\in\bigcap[\mathrm{Dom}(u,r,R)]^R\,;\qquad (9)}{\cfrac{r\in\bigcup_{p\in \mathrm{Dom}(u)}[p]_R\vdash \neg\neg\, r\in\bigcap[\mathrm{Dom}(u,r,R)]^R\ \&\ \&\ r\notin\bigcup(\mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R)}{\cfrac{r\in\bigcup_{p\in \mathrm{Dom}(u)}[p]_R\vdash\ \vdash \neg\neg\, r\in\bigcap[\mathrm{Dom}(u,r,R)]^R-\bigcup(\mathrm{Dom}(\mathrm{Sh}(u,R,S))-[\mathrm{Dom}(u,r,R)]^R)}{r\in\bigcup_{p\in \mathrm{Dom}(u)}[p]_R\vdash \neg\neg\, r\in[\mathrm{Dom}(u,r,R)]^R{}_{\frown\mathrm{Sh}(u,R,S)}}}} \qquad (10)$$

$$\langle p,\sigma\rangle \in u \vdash \neg\neg\, p \in \mathrm{Dom}(u);$$

$$\neg\neg\, p \in \mathrm{Dom}(u),\ r \in [p]_R \vdash \neg\neg\, p \in \mathrm{Dom}(u,r,R)$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg\neg\, p \in \mathrm{Dom}(u,r,R)\,;$$

$$\neg\neg\, p \in \mathrm{Dom}(u,r,R) \vdash \neg\neg\, [p]_R \in [\mathrm{Dom}(u,r,R)]^R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg\neg\, [p]_R \in [\mathrm{Dom}(u,r,R)]^R\,;$$

$$\neg\neg\, [p]_R \in [\mathrm{Dom}(u,r,R)]^R,$$

$$\neg\neg \langle [p]_R, \underline{[\check{\sigma}]_S}_R \rangle \in \mathrm{Sh}(u,R,S) \vdash \neg\neg\, \underline{[\check{\sigma}]_S}_R \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R,$$

$$\neg\neg \langle [p]_R, \underline{[\check{\sigma}]_S}_R \rangle \in \mathrm{Sh}(u,R,S) \vdash \neg\neg\, \underline{[\check{\sigma}]_S}_R \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R$$

$$\langle p,\sigma\rangle \in u \vdash \neg\neg \langle [p]_R, \underline{[\check{\sigma}]_S}_R \rangle \in \mathrm{Sh}(u,R,S)\,;$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg\neg\, \underline{[\check{\sigma}]_S}_R \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R\,;$$

$$\neg\neg\, \underline{[\check{\sigma}]_S}_R \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R \vdash$$

$$\vdash \llbracket \check{\delta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \subset' \bigcup_{v \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R} \llbracket \check{\delta} \in v \rrbracket_R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \llbracket \check{\delta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \subset' \bigcup_{v \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R} \llbracket \check{\delta} \in v \rrbracket_R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg_R \neg_R \llbracket \check{\delta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R \subset' \neg_R \neg_R \bigcup_{v \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R} \llbracket \check{\delta} \in v \rrbracket_R\,;$$

$$V^C(S) \vdash \neg_R \neg_R \llbracket \check{\delta} \in \underline{[\check{\sigma}]_S}_R \rrbracket_R = \llbracket \neg\neg\, \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R,\ V^C(S) \vdash$$

$$\vdash \llbracket \neg\neg\, \check{\delta} \in [\check{\sigma}]_S \rrbracket_R \subset' \neg_R \neg_R \bigcup_{v \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R} \llbracket \check{\delta} \in v \rrbracket_R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R,\ V^C(S),$$

$$r \in \llbracket \neg\neg\, \check{\delta} \in [\check{\sigma}]_S \rrbracket_R \vdash \neg\neg\, r \in \neg_R \neg_R \bigcup_{v \in \mathrm{Sh}(u,R,S)\,{}''[\mathrm{Dom}(u,r,R)]^R} \llbracket \check{\delta} \in v \rrbracket_R \qquad (11)$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg\neg\, p \in \mathrm{Dom}(u,r,R)\,;$$

$$\neg\neg\, p \in \mathrm{Dom}(u,r,R) \vdash \mathrm{Dom}(u,r,R) \neq 0$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \mathrm{Dom}(u,r,R) \neq 0\,;$$

$$(7) \Longrightarrow \quad \mathrm{Dom}(u,r,R) \neq 0 \vdash \neg\neg\, r \in \bigcap\, [\mathrm{Dom}(u,r,R)]^R$$

$$\langle p,\sigma\rangle \in u,\ r \in [p]_R \vdash \neg\neg\, r \in \bigcap\, [\mathrm{Dom}(u,r,R)]^R \qquad (12)$$

$$\dfrac{(12)\,; \qquad\qquad (10)}{\begin{array}{l}\langle p,\sigma\rangle\in u,\ r\in[p]_R,\ V^C(S),\ r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R\vdash\\ \vdash\neg\neg r\in\bigcap\,[\mathrm{Dom}(u,r,R)]^R\ \&\ \neg\neg r\in\overline{}_R\,\overline{}_R\bigcup\limits_{v\in\mathrm{Sh}(u,R,S)\,''[\mathrm{Dom}(u,r,R)]^R}[\![\check{\delta}\in v]\!]_R\end{array}}$$

$$\dfrac{\begin{array}{l}\langle p,\sigma\rangle\in u,\ r\in[p]_R,\ V^C(S),\ r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R\vdash\\ \vdash\neg\neg r\in\bigcap\,[\mathrm{Dom}(u,r,R)]^R\cap\overline{}_R\,\overline{}_R\bigcup\limits_{v\in\mathrm{Sh}(u,R,S)\,''[\mathrm{Dom}(u,r,R)]^R}[\![\check{\delta}\in v]\!]_R\,;\\ [\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{O}(R),\ [\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{Dom}(\mathrm{Sh}(u,R,S))\vdash\\ \qquad\vdash\bigcap\,[\mathrm{Dom}(u,r,R)]^R\cap\overline{}_R\,\overline{}_R\bigcup\limits_{v\in\mathrm{Sh}(u,R,S)\,''[\mathrm{Dom}(u,r,R)]^R}[\![\check{\delta}\in v]\!]_R\subset'\\ \qquad\subset'\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R \qquad \underset{[\mathrm{Dom}(u,r,R)]^R\ \ \check{\delta}\ \ \mathrm{Sh}(u,R,S)}{\overset{Z\ \ t\ \ S}{\Longleftarrow}} \qquad (6)\end{array}}{\begin{array}{l}\langle p,\sigma\rangle\in u,\ r\in[p]_R,\ V^C(S),\ r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R,\\ [\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{O}(R),\ [\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{Dom}(\mathrm{Sh}(u,R,S))\vdash\\ \vdash\neg\neg r\in\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R\end{array}}$$

$$\dfrac{\vdash[\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{O}(R)\,;}{\begin{array}{l}\langle p,\sigma\rangle\in u,\ r\in[p]_R,\ V^C(S),\ r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R,\\ [\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{Dom}(\mathrm{Sh}(u,R,S))\vdash\neg\neg r\in\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R\end{array}}$$

$$\dfrac{\vdash[\mathrm{Dom}(u,r,R)]^R\subset'\mathrm{Dom}(\mathrm{Sh}(u,R,S))\,;}{\begin{array}{l}\langle p,\sigma\rangle\in u,\ r\in[p]_R,\ V^C(S),\ r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R\vdash\\ \vdash\neg\neg r\in\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R\end{array}}$$

$$\dfrac{}{\begin{array}{l}\neg\neg r\in[p]_R\ \&\ \neg\neg r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R,\ \langle p,\sigma\rangle\in u,\ V^C(S)\vdash\\ \vdash\neg\neg r\in\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R\end{array}}$$

$$\dfrac{\langle r,\delta\rangle\in[\langle p,\sigma\rangle]_Q\vdash\neg\neg r\in[p]_R\ \&\ \neg\neg r\in[\![\neg\neg\check{\delta}\in[\check{\sigma}]_S]\!]_R\,;}{\begin{array}{l}\langle r,\delta\rangle\in[\langle p,\sigma\rangle]_Q,\ \langle p,\sigma\rangle\in u,\ V^C(S)\vdash\\ \vdash\neg\neg r\in\overline{}_R\,\overline{}_R[\![\check{\delta}\in\mathring{\mathrm{Sh}}(u,R,S)]\!]_R\end{array}} \qquad (13)$$

$$(13)\,; \quad \langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q \vdash \neg\neg\,\delta\in\Delta$$

$$\langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S)\vdash$$
$$\vdash \neg\neg\langle r,\delta\rangle \in {}_{\overline{R}}{}_{\overline{R}}\llbracket \check{\delta}\in \mathrm{Sh}\mathring{(}u,R,S)\rrbracket_R \times\Delta$$

$$\langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q \vdash \neg\neg\langle r,\delta\rangle\in[\langle r,\delta\rangle]_Q\,;$$

$$\langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S)\vdash$$
$$\vdash \neg\neg\langle r,\delta\rangle\in[\langle r,\delta\rangle]_Q\cap({}_{\overline{R}}{}_{\overline{R}}\llbracket \check{\delta}\in \mathrm{Sh}\mathring{(}u,R,S)\rrbracket_R \times\Delta\,)$$

$$\langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S)\vdash$$
$$\vdash \neg\neg\langle r,\delta\rangle\in \bigcup_{q\in|R|}\ \bigcup_{\eta\in\Delta}(\,[\langle q,\eta\rangle]_Q\cap({}_{\overline{R}}{}_{\overline{R}}\llbracket \check{\eta}\in \mathrm{Sh}\mathring{(}u,R,S)\rrbracket_R \times\Delta\,))$$

$$\langle r,\delta\rangle \in [\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S), t=\langle r,\delta\rangle\vdash$$
$$\vdash \neg\neg\, t\in \bigcup_{q\in|R|}\ \bigcup_{\eta\in\Delta}(\,[\langle q,\eta\rangle]_Q\cap({}_{\overline{R}}{}_{\overline{R}}\llbracket \check{\eta}\in \mathrm{Sh}\mathring{(}u,R,S)\rrbracket_R \times\Delta\,))$$

$$t=\langle r,\delta\rangle, t\in[\langle p,\sigma\rangle]_Q\vdash\langle r,\delta\rangle\in[\langle p,\sigma\rangle]_Q\,;$$

$$t=\langle r,\delta\rangle, t\in[\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S)\vdash\neg\neg\, t\in\ldots$$

$$t=\langle r,\delta\rangle, t\in[\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S), t\notin\ldots\vdash$$

$$\neg\neg\exists r,\delta(t=\langle r,\delta\rangle), t\in[\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S), t\notin\ldots\vdash$$

$$t\in[\langle p,\sigma\rangle]_Q\vdash\neg\neg\exists r,\delta(t=\langle r,\delta\rangle)\,;$$

$$t\in[\langle p,\sigma\rangle]_Q ,\langle p,\sigma\rangle\in u, V^C(S), t\notin\ldots\vdash$$

$$\neg\neg\, t\in[\langle p,\sigma\rangle]_Q\ \&\ \langle p,\sigma\rangle\in u, V^C(S), t\notin\ldots\vdash$$

$$y=\langle p,\sigma\rangle, t\in[y]_Q, y\in u\vdash\neg\neg\, t\in[\langle p,\sigma\rangle]_Q\ \&\ \langle p,\sigma\rangle\in u\,;$$

$$y=\langle p,\sigma\rangle, t\in[y]_Q, y\in u, V^C(S), t\notin\ldots\vdash$$

$$\neg\neg\exists p,\sigma(y=\langle p,\sigma\rangle), t\in[y]_Q, y\in u, V^C(S), t\notin\ldots\vdash$$

$$y\in u, u\subset'|R|\times\Delta\vdash\neg\neg\exists p,\sigma(y=\langle p,\sigma\rangle)\,;$$

$$y\in u, u\subset'|R|\times\Delta\,, t\in[y]_Q, V^C(S), t\notin\ldots\vdash$$

$$\neg\neg\exists y\in u\,(t\in[y]_Q), u\subset'|R|\times\Delta\,, V^C(S), t\notin\ldots\vdash$$

$$t\in\bigcup_{y\in u}[y]_Q\,, u\subset'|R|\times\Delta\,, V^C(S), t\notin\ldots\vdash$$

$$u\subset'|R|\times\Delta, V^C(S)\vdash$$
$$\vdash\bigcup_{y\in u}[y]_Q\subset'\bigcup_{q\in|R|}\ \bigcup_{\eta\in\Delta}(\,[\langle q,\eta\rangle]_Q\cap {}_{\overline{R}}{}_{\overline{R}}\llbracket \check{\eta}\in \mathrm{Sh}\mathring{(}u,R,S)\rrbracket_R \times\Delta\,) \qquad (14)$$

$$r \in |R| \vdash \neg\neg\langle r, [r]_R \rangle \in \underset{\smile}{R}\,;$$

$$\neg\neg\langle r, [r]_R \rangle \in \underset{\smile}{R} \vdash \neg\neg\langle [r]_R, r \rangle \in \underset{\smile}{R}^{-1}$$

$$r \in |R| \vdash \neg\neg\langle [r]_R, r \rangle \in \underset{\smile}{R}^{-1}\,;$$

$$\neg\neg\langle [r]_R, r \rangle \in \underset{\smile}{R}^{-1}, [r]_R \in Z \vdash \neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z$$

$$r \in |R|, [r]_R \in Z \vdash \neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z$$

$$[r]_R \in Z, r \in |R| \vdash \neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z$$

$$t \in Z, t = [r]_R \vdash [r]_R \in Z\,;$$

$$t \in Z, t = [r]_R, r \in |R| \vdash \neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z\,;$$

$$\langle r, \eta \rangle \in u \vdash \neg\neg \eta \in u[r]$$

$$t \in Z, t = [r]_R, r \in |R|, \langle r, \eta \rangle \in u \vdash \neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z \;\&\; \neg\neg \eta \in u[r]\,;$$

$$\neg\neg r \in \underset{\smile}{R}^{-1}\text{”}Z \;\&\; \neg\neg \eta \in u[r],$$

$$v = \underline{[\check{\eta}]}_{SR} \vdash \neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR})$$

$$t \in Z, t = [r]_R, r \in |R|, \langle r, \eta \rangle \in u, v = \underline{[\check{\eta}]}_{SR} \vdash$$

$$\vdash \neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR})$$

$$t \in Z, t = [r]_R, r \in |R|, \langle r, \eta \rangle \in u, v = \underline{[\check{\eta}]}_{SR},$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash$$

$$\neg\neg r \in |R|, t \in Z, t = [r]_R, \langle r, \eta \rangle \in u, v = \underline{[\check{\eta}]}_{SR},$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash$$

$$\langle r, \eta \rangle \in u, u \subset' |R| \times \Delta \vdash \neg\neg r \in |R|\,;$$

$$t \in Z, t = [r]_R, \langle r, \eta \rangle \in u, u \subset' |R| \times \Delta, v = \underline{[\check{\eta}]}_{SR},$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash$$

$$t = [r]_R \;\&\; v = \underline{[\check{\eta}]}_{SR}, \langle r, \eta \rangle \in u, t \in Z, u \subset' |R| \times \Delta,$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash$$

$$\langle t, v \rangle = \langle [r]_R, \underline{[\check{\eta}]}_{SR} \rangle \vdash t = [r]_R \;\&\; v = \underline{[\check{\eta}]}_{SR}\,;$$

$$\langle t, v \rangle = \langle [r]_R, \underline{[\check{\eta}]}_{SR} \rangle, \langle r, \eta \rangle \in u, t \in Z, u \subset' |R| \times \Delta,$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash$$

$$\neg\neg\exists r, \eta\,(\langle r, \eta \rangle \in u \;\&\; \langle t, v \rangle = \langle [r]_R, \underline{[\check{\eta}]}_{SR} \rangle), t \in Z, u \subset' |R| \times \Delta,$$

$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z \;\; \exists \sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}) \vdash \qquad (15)$$

Схема 2.3.10

$$(15)$$

$$\langle t, v \rangle \in \mathrm{Sh}(u,R,S),\ t \in Z, u \subset' |R| \times \Delta,$$
$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R}) \vdash$$

$$\langle t, v \rangle \in \mathrm{Sh}(u,R,S),\ t \in Z, u \subset' |R| \times \Delta \vdash$$
$$\vdash \neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R})$$

$$\langle t, v \rangle \in \mathrm{Sh}(u,R,S),\ t \in Z, u \subset' |R| \times \Delta,$$
$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R}) \vdash$$

$$\neg\neg\exists t \in Z\, \exists v\, (\langle t, v \rangle \in \mathrm{Sh}(u,R,S)), u \subset' |R| \times \Delta,$$
$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R}) \vdash$$

$$v \in \mathrm{Sh}(u,R,S)\text{”}Z, u \subset' |R| \times \Delta,$$
$$\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R}) \vdash$$

$$v \in \mathrm{Sh}(u,R,S)\text{”}Z, u \subset' |R| \times \Delta \vdash$$
$$\vdash \neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{”}Z\ \ \exists\sigma \in u[p]\,(v = [\check{\sigma}]_{S_R}) \qquad (16)$$

$$\langle p, u \rangle = \langle r, [r]_R \rangle \vdash p = r \,\&\, u = [r]_R\,;$$
$$p = r \,\&\, u = [r]_R \vdash u = [p]_R$$

$$\langle p, u \rangle = \langle r, [r]_R \rangle \vdash u = [p]_R\,;$$
$$u = [p]_R,\ u \in Z \vdash [p]_R \in Z$$

$$\langle p, u \rangle = \langle r, [r]_R \rangle, u \in Z \vdash [p]_R \in Z$$

$$\langle p, u \rangle = \langle r, [r]_R \rangle, u \in Z, [p]_R \notin Z \vdash$$

$$\neg\neg\exists r \in |R|\,(\langle p, u \rangle = \langle r, [r]_R \rangle), u \in Z, [p]_R \notin Z \vdash$$
$$\langle p, u \rangle \in \underset{\smile}{R} \vdash \neg\neg\exists r \in |R|\,(\langle p, u \rangle = \langle r, [r]_R \rangle)\,;$$

$$\langle p, u \rangle \in \underset{\smile}{R}, u \in Z, [p]_R \notin Z \vdash$$

$$\neg\neg\langle p, u \rangle \in \underset{\smile}{R}, u \in Z, [p]_R \notin Z \vdash$$
$$\langle u, p \rangle \in \underset{\smile}{R}^{-1} \vdash \neg\neg\langle p, u \rangle \in \underset{\smile}{R}\,;$$

$$\langle u, p \rangle \in \underset{\smile}{R}^{-1}, u \in Z, [p]_R \notin Z \vdash$$

$$\neg\neg\exists u \in Z\,(\langle u, p \rangle \in \underset{\smile}{R}^{-1}), [p]_R \notin Z \vdash$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z, [p]_R \notin Z \vdash$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z \vdash \neg\neg[p]_R \in Z \qquad (17)$$

$$r \in \bigcap Z \vdash \forall v \in Z\,(\neg\neg r \in v)$$

$$[p]_R \in Z, r \in \bigcap Z \vdash \neg\neg r \in [p]_R$$

$$\neg\neg [p]_R \in Z, r \in \bigcap Z \vdash \neg\neg r \in [p]_R$$

$$(17) \Longrightarrow \quad p \in \underset{\smile}{R}^{-1}\text{”}Z \vdash \neg\neg [p]_R \in Z\,;$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z, r \in \bigcap Z \vdash \neg\neg r \in [p]_R \qquad (18)$$

$$\vdash |R| \subset' \llbracket \mathrm{Tv}(S) \rrbracket_R\,;$$

$$V^C(S) \vdash \llbracket \mathrm{Tv}(S) \rrbracket_R \cap \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R \cap \llbracket \neg\neg \check{\eta} \in [\check{\sigma}]_S \rrbracket_R \subset' \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$V^C(S) \vdash \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R \cap \llbracket \neg\neg \check{\eta} \in [\check{\sigma}]_S \rrbracket_R \subset' \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$V^C(S) \vdash \llbracket \check{\eta} \in \underline{[\check{\sigma}]}_{SR} \rrbracket_R \subset' \llbracket \neg\neg \check{\eta} \in [\check{\sigma}]_S \rrbracket_R\,;$$

$$V^C(S) \vdash \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R \cap \llbracket \check{\eta} \in \underline{[\check{\sigma}]}_{SR} \rrbracket_R \subset' \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$\neg\neg r \in \llbracket \check{\eta} \in \underline{[\check{\sigma}]}_{SR} \rrbracket_R,\ r \in \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R,\ V^C(S) \vdash \neg\neg r \in \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$v = \underline{[\check{\sigma}]}_{SR},\ r \in \llbracket \check{\eta} \in v \rrbracket_R \vdash \neg\neg r \in \llbracket \check{\eta} \in \underline{[\check{\sigma}]}_{SR} \rrbracket_R\,;$$

$$v = \underline{[\check{\sigma}]}_{SR},\ r \in \llbracket \check{\eta} \in v \rrbracket_R,\ r \in \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R,\ V^C(S) \vdash$$
$$\vdash \neg\neg r \in \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R$$

$$(18) \Longrightarrow \quad p \in \underset{\smile}{R}^{-1}\text{”}Z, r \in \bigcap Z \vdash \neg\neg r \in [p]_R\,;$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z, r \in \bigcap Z, v = \underline{[\check{\sigma}]}_{SR},\ r \in \llbracket \check{\eta} \in v \rrbracket_R,$$
$$r \in \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R,\ V^C(S) \vdash$$
$$\vdash \neg\neg r \in [p]_R \ \&\ \neg\neg r \in \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R\,;$$

$$\neg\neg r \in [p]_R \ \&\ \neg\neg r \in \llbracket \neg\neg \check{\delta} \in [\check{\sigma}]_S \rrbracket_R,\quad \delta, \sigma \in \Delta \vdash$$
$$\vdash \neg\neg \langle r, \delta \rangle \in [\langle p, \sigma \rangle]_Q$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z, r \in \bigcap Z, v = \underline{[\check{\sigma}]}_{SR},\ r \in \llbracket \check{\eta} \in v \rrbracket_R,$$
$$r \in \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R,\ V^C(S),\ \delta, \sigma \in \Delta \vdash \neg\neg \langle r, \delta \rangle \in [\langle p, \sigma \rangle]_Q\,;$$

$$\neg\neg \langle r, \delta \rangle \in [\langle p, \sigma \rangle]_Q,$$
$$t = \langle r, \delta \rangle \vdash \neg\neg t \in [\langle p, \sigma \rangle]_Q$$

$$p \in \underset{\smile}{R}^{-1}\text{”}Z, r \in \bigcap Z, v = \underline{[\check{\sigma}]}_{SR},\ r \in \llbracket \check{\eta} \in v \rrbracket_R,$$
$$r \in \llbracket \neg\neg \check{\delta} \in [\check{\eta}]_S \rrbracket_R,\ V^C(S),\ \delta, \sigma \in \Delta,\ t = \langle r, \delta \rangle \vdash$$
$$\vdash \neg\neg t \in [\langle p, \sigma \rangle]_Q \qquad (19)$$

Схема 2.3.12

$$(19)\,;$$

$$\neg\neg\, t \in [\langle p,\sigma\rangle]_Q\,,\langle p,\sigma\rangle \in u \vdash \neg\neg\, t \in \bigcup_{y\in u}\,[y]_Q$$

$$p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\;\; \delta,\sigma \in \Delta,\, t = \langle r,\delta\rangle,\langle p,\sigma\rangle \in u \vdash$$
$$\vdash \neg\neg\, t \in \bigcup_{y\in u}\,[y]_Q$$

$$p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\;\; \delta,\sigma \in \Delta,\, t = \langle r,\delta\rangle,\langle p,\sigma\rangle \in u \vdash$$
$$t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$\neg\neg\,\sigma \in \Delta,\, p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\langle p,\sigma\rangle \in u \vdash$$
$$t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$\langle p,\sigma\rangle \in u,\, u \subset' |R| \times \Delta \vdash \neg\neg\,\sigma \in \Delta\,;$$

$$\langle p,\sigma\rangle \in u,\, u \subset' |R| \times \Delta,\, p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\, t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$\neg\neg\,\langle p,\sigma\rangle \in u,\, u \subset' \ldots,\, p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\, t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$\sigma \in u[p]\,,\, u \subset' |R| \times \Delta,\, p \in \underset{\smile}{R}^{-1}\text{"}Z,\, r \in \bigcap Z,\, v = \underline{[\check{\sigma}]}_{SR}\,,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\, t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$\neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{"}Z\;\; \exists\sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR}),$$
$$u \subset' |R| \times \Delta,\, r \in \bigcap Z,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\, t \notin \bigcup_{y\in u}\,[y]_Q \vdash$$

$$(16) \Longrightarrow \quad v \in \mathrm{Sh}(u,R,S)\text{"}Z,\, u \subset' |R| \times \Delta \vdash$$
$$\vdash \neg\neg\exists p \in \underset{\smile}{R}^{-1}\text{"}Z\;\; \exists\sigma \in u[p]\,(v = \underline{[\check{\sigma}]}_{SR})\,;$$

$$v \in \mathrm{Sh}(u,R,S)\text{"}Z,\, u \subset' |R| \times \Delta,\, r \in \bigcap Z,\, r \in \llbracket \check{\eta} \in v \rrbracket_R\,,$$
$$r \in \llbracket \neg\neg\,\check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle r,\delta\rangle,\, t \notin \bigcup_{y\in u}\,[y]_Q \qquad (20)$$

Схема 2.3.13

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad (20) \\
\hline
\neg\neg\exists v \in \mathrm{Sh}(u,R,S)\text{”}Z\,(r \in [\![\breve{\eta} \in v]\!]_R),\, u \subset' |R| \times \Delta,\, r \in \bigcap Z, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash \\
\hline
\neg\neg\, r \in \bigcup\limits_{v \in \mathrm{Sh}(u,R,S)\text{”}Z} [\![\breve{\eta} \in v]\!]_R\,,\, u \subset' |R| \times \Delta,\, r \in \bigcap Z, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash
\end{array}
\qquad (21)
$$

$$
\begin{array}{l}
(3) \Longrightarrow \quad Z_{\frown S} \cap [\![t \in \mathring{S}]\!]_R \subset' \bigcup\limits_{v \in S\text{”}Z} [\![t \in v]\!]_R \;\overset{S\quad t}{\underset{\mathrm{Sh}(u,R,S)\quad \breve{\eta}}{\Longrightarrow}} \\
\quad \overset{S\quad t}{\underset{\mathrm{Sh}(u,R,S)\quad \breve{\eta}}{\Longrightarrow}} \quad Z_{\frown \mathrm{Sh}(u,R,S)} \cap [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R \subset' \bigcup\limits_{v \in \mathrm{Sh}(u,R,S)\text{”}Z} [\![\breve{\eta} \in v]\!]_R \\
\hline
r \in Z_{\frown \mathrm{Sh}(u,R,S)}\,,\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R \vdash \neg\neg\, r \in \bigcup\limits_{v \in \mathrm{Sh}(u,R,S)\text{”}Z} [\![\breve{\eta} \in v]\!]_R\,; \\
\qquad\qquad\qquad\qquad (21) \\
\hline
r \in Z_{\frown \mathrm{Sh}(u,R,S)}\,,\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R\,,\, u \subset' |R| \times \Delta,\, r \in \bigcap Z, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash \\
\hline
r \in Z_{\frown \mathrm{Sh}(u,R,S)}\,,\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R\,,\, u \subset' |R| \times \Delta,\, \neg\neg\, r \in \bigcap Z, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash \\
\hline
r \in Z_{\frown \mathrm{Sh}(u,R,S)}\,,\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R\,,\, u \subset' |R| \times \Delta, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash \\
\hline
\neg\neg\exists Z\,(r \in Z_{\frown \mathrm{Sh}(u,R,S)}),\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R\,,\, u \subset' |R| \times \Delta, \\
r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash \\
(10) \Rightarrow r \in \bigcup\limits_{p \in \mathrm{Dom}(u)} [p]_R \vdash \neg\neg\exists Z\,(r \in Z_{\frown \mathrm{Sh}(u,R,S)})\,; \\
\hline
\qquad r \in \bigcup\limits_{p \in \mathrm{Dom}(u)} [p]_R\,,\, r \in [\![\breve{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)]\!]_R\,,\, u \subset' |R| \times \Delta, \\
\; r \in [\![\neg\neg\,\check{\delta} \in [\breve{\eta}]_S]\!]_R\,,\, V^C(S),\, \delta \in \Delta,\, t = \langle\, r, \delta\,\rangle\,,\, t \notin \bigcup\limits_{y \in u} [y]_Q \vdash
\end{array}
\qquad (22)
$$

Схема 2.3.14

$$(22)$$

$$\neg\neg\, r \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \;\&\; \neg\neg\, \delta \in \Delta,\; \neg\neg\, r \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R\,,\; u \subset' |R| \times \Delta,$$
$$\neg\neg\, r \in \llbracket \neg\neg\, \check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\; V^C(S),\; t = \langle\, r, \delta \,\rangle,\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\langle\, r, \delta \,\rangle \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; \langle\, r, \delta \,\rangle \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta,\; u \subset' |R| \times \Delta,$$
$$\neg\neg\, r \in \llbracket \neg\neg\, \check{\delta} \in [\check{\eta}]_S \rrbracket_R\,,\; V^C(S),\; t = \langle\, r, \delta \,\rangle,\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$
$$\langle\, r, \delta \,\rangle \in [\langle q, \eta \,\rangle]_Q \vdash \neg\neg\, r \in \llbracket \neg\neg\, \check{\delta} \in [\check{\eta}]_S \rrbracket_R\,;$$

$$\langle\, r, \delta \,\rangle \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; \langle\, r, \delta \,\rangle \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta,\; u \subset' |R| \times \Delta,$$
$$\langle\, r, \delta \,\rangle \in [\langle q, \eta \,\rangle]_Q\,,\; V^C(S),\; t = \langle\, r, \delta \,\rangle,\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$t = \langle\, r, \delta \,\rangle,\; t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; t \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta,\; u \subset' |R| \times \Delta,$$
$$t \in [\langle q, \eta \,\rangle]_Q\,,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\neg\neg \exists r, \delta\, (t = \langle\, r, \delta \,\rangle),\; t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; \ldots \vdash$$

$$t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; t \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta,\; u \subset' |R| \times \Delta,$$
$$t \in [\langle q, \eta \,\rangle]_Q\,,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\neg\neg\, t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; \neg\neg\, t \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta,\; u \subset' |R| \times \Delta,$$
$$\neg\neg\, t \in [\langle q, \eta \,\rangle]_Q\,,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\neg\neg\, t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; \neg\neg\, t \in [\langle q, \eta \,\rangle]_Q \;\&\; \neg\neg\, t \in \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta$$
$$u \subset' |R| \times \Delta,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\neg\neg\, t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; t \in [\langle q, \eta \,\rangle]_Q \cap \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta$$
$$u \subset' |R| \times \Delta,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash$$

$$\neg\neg \exists q \in |R|\, \exists \eta \in \Delta\, (\, t \in [\langle q, \eta \,\rangle]_Q \cap \llbracket \check{\eta} \in \mathring{\mathrm{Sh}}(u,R,S)\rrbracket_R \times \Delta\,),$$
$$\neg\neg\, t \in \bigcup_{p\in \mathrm{Dom}(u)} [p]_R \times \Delta,\; u \subset' |R| \times \Delta,\; V^C(S),\; t \notin \bigcup_{y\in u} [y]_Q \vdash \qquad (23)$$

Схема 2.3.15

$$\frac{\begin{array}{l}\neg\neg\exists q \in |R|\, \exists \eta \in \Delta\, (t \in [\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u}, R, S)]\!]_R \times \Delta),\\ \neg\neg\, t \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta,\, u \subset' |R| \times \Delta,\, V^C(S),\, t \notin \bigcup_{y \in u} [y]_Q \vdash \Longleftarrow \ (23)\end{array}}{\begin{array}{l}\neg\neg\, t \in \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u}, R, S)]\!]_R \times \Delta),\\ \neg\neg\, t \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta,\, u \subset' |R| \times \Delta,\, V^C(S),\, t \notin \bigcup_{y \in u} [y]_Q \vdash\end{array}}$$

$$\frac{\begin{array}{l} t \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \ \cap \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u}, R, S)]\!]_R \times \Delta),\\ u \subset' |R| \times \Delta,\, V^C(S),\, t \notin \bigcup_{y \in u} [y]_Q \vdash\end{array}}{\begin{array}{l} u \subset' |R| \times \Delta,\, V^C(S) \vdash \neg\exists t \in \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \ \cap \\ \cap \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u}, R, S)]\!]_R \times \Delta)\, (t \notin \bigcup_{y \in u} [y]_Q)\end{array}}$$

$$\begin{array}{l} u \subset' |R| \times \Delta,\, V^C(S) \vdash \\ \vdash \bigcup_{p \in \mathrm{Dom}(u)} [p]_R \times \Delta \ \cap \bigcup_{q \in |R|} \bigcup_{\eta \in \Delta} ([\langle q, \eta \rangle]_Q \cap [\![\check{\eta} \in \mathrm{Sh}(\mathring{u}, R, S)]\!]_R \times \Delta) \subset' \\ \subset' \bigcup_{y \in u} [y]_Q \qquad (24)\end{array}$$

Схема 2.3.16

$$\langle r,\sigma\rangle\in\mathfrak{S}\vdash\neg\neg\langle[r]_R,\underline{[\check{\sigma}]}_{S_R}\rangle\in\mathrm{Sh}(\mathfrak{S},R,S)$$

$$\langle r,\sigma\rangle\in\mathfrak{S}\vdash\neg\neg\underline{[\check{\sigma}]}_{S_R}\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R];$$

$$\neg\neg\underline{[\check{\sigma}]}_{S_R}\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R],\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash$$

$$\vdash\neg\neg r\in\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R]}[\![t\in v]\!]_R$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash\neg\neg r\in\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R]}[\![t\in v]\!]_R$$

$$r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash\neg\neg r\in[r]_R;$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash\neg\neg r\in[r]_R\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R]}[\![t\in v]\!]_R;$$

$$[r]_R\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))\vdash[r]_R\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[[r]_R]}[\![t\in v]\!]_R\subset'$$

$$\subset'\bigcup_{u\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))}(u\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[u]}[\![t\in v]\!]_R)$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,\ [r]_R\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))\vdash$$

$$\vdash\neg\neg r\in\bigcup_{u\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))}(u\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[u]}[\![t\in v]\!]_R)$$

$$\neg\neg[r]_R\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S)),\ \langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash$$

$$\vdash\neg\neg r\in\bigcup_{u\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))}(u\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[u]}[\![t\in v]\!]_R)$$

$$\langle r,\sigma\rangle\in\mathfrak{S}\vdash\neg\neg[r]_R\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S));$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash$$

$$\vdash\neg\neg r\in\bigcup_{u\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))}(u\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[u]}[\![t\in v]\!]_R);$$

$$\vdash[\![t\in\mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R=\bigcup_{u\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{S},R,S))}(u\cap\bigcup_{v\in\mathrm{Sh}(\mathfrak{S},R,S)[u]}[\![t\in v]\!]_R)\ \overset{S}{\underset{\mathrm{Sh}(\mathfrak{S},R,S)}{\Longleftarrow}}\quad(1)$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R\vdash\neg\neg r\in[\![t\in\mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R$$

$$\langle r,\sigma\rangle\in\mathfrak{S},\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,\ r\notin[\![t\in\mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R\vdash$$

$$r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,\ r\notin[\![t\in\mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R\vdash\langle r,\sigma\rangle\notin\mathfrak{S}\quad(25)$$

Схема 2.3.17

$$r \in [\![u = v]\!]_R,\ \sigma \in \Delta \vdash \neg\neg\langle r,\sigma\rangle \in [\![u = v]\!]_R \times \Delta;$$
$$V^C(u), V^C(v) \vdash [\![u = v]\!]_R \times \Delta = f^{-1}{}'' [\![u = v]\!]_R = [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q$$

$$r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v) \vdash \neg\neg\langle r,\sigma\rangle \in [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q;$$
$$\neg\neg\langle r,\sigma\rangle \in [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q,$$
$$\langle r,\sigma\rangle \in \mathfrak{A} \vdash \neg\neg\langle r,\sigma\rangle \in [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A}$$

$$r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v), \langle r,\sigma\rangle \in \mathfrak{A} \vdash$$
$$\vdash \neg\neg\langle r,\sigma\rangle \in [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A};$$
$$\neg\neg\langle r,\sigma\rangle \in [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S}, \langle r,\sigma\rangle \notin \mathfrak{S} \vdash$$

$$r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v), \langle r,\sigma\rangle \in \mathfrak{A},$$
$$[\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S}, \langle r,\sigma\rangle \notin \mathfrak{S} \vdash$$

$$\langle r,\sigma\rangle \notin \mathfrak{S}, r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v), \langle r,\sigma\rangle \in \mathfrak{A},$$
$$[\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash$$
$$(25) \Longrightarrow\ r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,\ r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R \vdash \langle r,\sigma\rangle \notin \mathfrak{S};$$

$$r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,\ r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R,\ r \in [\![u = v]\!]_R,\ \sigma \in \Delta,$$
$$V^C(u), V^C(v), \langle r,\sigma\rangle \in \mathfrak{A}, [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash$$

$$\neg\neg\langle r,\sigma\rangle \in \mathfrak{A}, r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,\ r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R,\ r \in [\![u = v]\!]_R,$$
$$\sigma \in \Delta, V^C(u), V^C(v), [\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash$$
$$\langle r,\sigma\rangle \in [\langle p,\sigma\rangle]_Q,\ \langle p,\sigma\rangle \in \mathfrak{A} \in \mathrm{O}(Q) \vdash \neg\neg\langle r,\sigma\rangle \in \mathfrak{A};$$

$$\langle r,\sigma\rangle \in [\langle p,\sigma\rangle]_Q,\ \langle p,\sigma\rangle \in \mathfrak{A} \in \mathrm{O}(Q), r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,$$
$$r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R,\ r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v),$$
$$[\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash$$

$$\neg\neg\langle r,\sigma\rangle \in [\langle p,\sigma\rangle]_Q,\ \langle p,\sigma\rangle \in \mathfrak{A} \in \mathrm{O}(Q), r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,$$
$$r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R,\ r \in [\![u = v]\!]_R,\ \sigma \in \Delta,\ V^C(u), V^C(v),$$
$$[\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash$$
$$r \in [p]_R,\ \sigma \in \Delta \vdash \neg\neg\langle r,\sigma\rangle \in [\langle p,\sigma\rangle]_Q;$$

$$r \in [p]_R,\ \sigma \in \Delta,\ \langle p,\sigma\rangle \in \mathfrak{A} \in \mathrm{O}(Q), r \in [\![t \in \underline{[\check{\sigma}]}_{SR}]\!]_R,$$
$$r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S}, R, S)]\!]_R,\ r \in [\![u = v]\!]_R,\ V^C(u), V^C(v),$$
$$[\![f^{-1\frown}u = f^{-1\frown}v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash \qquad (26)$$

(26)

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$$\neg\neg\sigma\in\Delta,\ r\in[p]_R,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$
$$\langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q)\vdash\neg\neg\sigma\in\Delta;$$

***

$$r\in[p]_R,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

***

$$\neg\neg r\in[p]_R,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$
$$y=[p]_R,\ r\in y\vdash\neg\neg r\in[p]_R;$$

***

$$y=[p]_R,\ r\in y,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),\ r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

***

$$\neg\neg r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R,\ y=[p]_R,\ r\in y,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$
$$z=\underline{[\check{\sigma}]}_{S_R},\ r\in[\![t\in z]\!]_R\vdash\neg\neg r\in[\![t\in\underline{[\check{\sigma}]}_{S_R}]\!]_R;$$

***

$$z=\underline{[\check{\sigma}]}_{S_R},\ r\in[\![t\in z]\!]_R,\ y=[p]_R,\ r\in y,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

***

$$y=[p]_R\ \&\ z=\underline{[\check{\sigma}]}_{S_R},\ r\in[\![t\in z]\!]_R,\ r\in y,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$
$$\langle y,z\rangle=\langle[p]_R,\underline{[\check{\sigma}]}_{S_R}\rangle\vdash y=[p]_R\ \&\ z=\underline{[\check{\sigma}]}_{S_R};$$

***

$$\langle y,z\rangle=\langle[p]_R,\underline{[\check{\sigma}]}_{S_R}\rangle,\ r\in[\![t\in z]\!]_R,\ r\in y,\ \langle p,\sigma\rangle\in\mathfrak{A}\in\mathrm{O}(Q),$$
$$r\notin[\![t\in\mathrm{Sh}\mathring{(}\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash \qquad (27)$$

(27)

$$\neg\neg\exists p,\sigma\,(\langle p,\sigma\rangle\in\mathfrak{A}\ \&\ \langle y,z\rangle=\langle[p]_R,\underline{[\check{\sigma}]_{SR}}\rangle),\ r\in[\![t\in z]\!]_R,\ r\in y,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$\neg\neg\langle y,z\rangle\in\mathrm{Sh}(\mathfrak{A},R,S),\ r\in[\![t\in z]\!]_R,\ r\in y,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$z\in\mathrm{Sh}(\mathfrak{A},R,S)[y],\ r\in[\![t\in z]\!]_R,\ r\in y,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$\neg\neg\exists z\in\mathrm{Sh}(\mathfrak{A},R,S)[y]\,(r\in[\![t\in z]\!]_R),\ r\in y,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$r\in\bigcup_{z\in\mathrm{Sh}(\mathfrak{A},R,S)[y]}[\![t\in z]\!]_R,\ r\in y,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$\neg\neg r\in y\ \&\ \neg\neg r\in\bigcup_{z\in\mathrm{Sh}(\mathfrak{A},R,S)[y]}[\![t\in z]\!]_R,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$r\in y\cap\bigcup_{z\in\mathrm{Sh}(\mathfrak{A},R,S)[y]}[\![t\in z]\!]_R,$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash$$

$$\neg\neg\exists y\in\mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))\,(r\in y\cap\bigcup_{z\in\mathrm{Sh}(\mathfrak{A},R,S)[y]}[\![t\in z]\!]_R),$$
$$\mathfrak{A}\in\mathrm{O}(Q),\ r\notin[\![t\in\overset{\circ}{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\ r\in[\![u=v]\!]_R,\ V^C(u),\ V^C(v),$$
$$[\![f^{-1}{}^\frown u=f^{-1}{}^\frown v]\!]_Q\cap\mathfrak{A}\subset'\mathfrak{S}\vdash \qquad (28)$$

$$\frac{(28)}{\neg\neg r \in \bigcup_{y\in \mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))} (y \cap \bigcup_{z\in \mathrm{Sh}(\mathfrak{A},R,S)[y]} [\![t \in z]\!]_R),}$$

$$\frac{\begin{array}{l}\mathfrak{A} \in \mathrm{O}(Q), r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R, r \in [\![u = v]\!]_R, V^C(u), V^C(v),\\ \quad [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\\ \quad [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R = \bigcup_{y\in \mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))} (y \cap \bigcup_{z\in \mathrm{Sh}(\mathfrak{A},R,S)[y]} [\![t \in z]\!]_R),\\ r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R \vdash \neg\neg r \in \bigcup_{y\in \mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))} (y \cap \bigcup_{z\in \mathrm{Sh}(\mathfrak{A},R,S)[y]} [\![t \in z]\!]_R)\end{array}}{\begin{array}{l}\quad [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R = \bigcup_{y\in \mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))} (y \cap \bigcup_{z\in \mathrm{Sh}(\mathfrak{A},R,S)[y]} [\![t \in z]\!]_R),\\ r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R, \mathfrak{A} \in \mathrm{O}(Q), r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R, r \in [\![u = v]\!]_R,\\ V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\end{array}}$$

$$(1) \underset{\mathrm{Sh}(\mathfrak{A},R,S)}{\overset{S}{\Longrightarrow}} [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R = \bigcup_{y\in \mathrm{Dom}(\mathrm{Sh}(\mathfrak{A},R,S))} (y \cap \bigcup_{z\in \mathrm{Sh}(\mathfrak{A},R,S)[y]} [\![t \in z]\!]_R);$$

$$\frac{\begin{array}{l}r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R, \mathfrak{A} \in \mathrm{O}(Q), r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R, r \in [\![u = v]\!]_R,\\ V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\end{array}}{\begin{array}{l}\neg\neg r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R \,\&\, \neg\neg r \in [\![u = v]\!]_R, r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\\ \quad \mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\end{array}}$$

$$\frac{\begin{array}{l}r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R \cap [\![u = v]\!]_R, r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R,\\ \mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\end{array}}{\begin{array}{l}\mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\\ \vdash \neg\exists r \in [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R \cap [\![u = v]\!]_R (r \notin [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R)\end{array}}$$

$$\begin{array}{l}\mathfrak{A} \in \mathrm{O}(Q), V^C(u), V^C(v), [\![f^{-1}{}^\frown u = f^{-1}{}^\frown v]\!]_Q \cap \mathfrak{A} \subset' \mathfrak{S} \vdash\\ \vdash [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{A},R,S)]\!]_R \cap [\![u = v]\!]_R \subset' [\![t \in \mathring{\mathrm{Sh}}(\mathfrak{S},R,S)]\!]_R\end{array} \qquad (29)$$

Схема 2.4.1

$$V^C(t), V^D(u), V^C(S) \vdash \llbracket t \in \hat{u} \rrbracket_R = \bigcup_{r \in \mathrm{Dom}(\hat{u})} (\hat{u}(r) \cap \llbracket t = r \rrbracket_R) =$$

$$= \bigcup_{r \in \mathrm{Dom}(\hat{u})} (\bigcup_{v \in \mathrm{Dom}(u)} (\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket r = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \rangle \right\rrbracket_R) \cap$$

$$\cap \llbracket t = r \rrbracket_R) =$$

$$= \bigcup_{r \in \mathrm{Dom}(\hat{u})} \bigcup_{v \in \mathrm{Dom}(u)} (\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket r = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \rangle \right\rrbracket_R \cap$$

$$\cap \llbracket t = r \rrbracket_R) =$$

$$= \bigcup_{r \in \mathrm{Dom}(\hat{u})} \bigcup_{v \in \mathrm{Dom}(u)} (\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket t = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \rangle \right\rrbracket_R \cap$$

$$\cap \llbracket t = r \rrbracket_R) =$$

$$= \bigcup_{v \in \mathrm{Dom}(u)} (\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket t = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \rangle \right\rrbracket_R \cap$$

$$\cap \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket t = r \rrbracket_R) =$$

$$V^C(t), V^D(u), V^C(S) \vdash \llbracket t \in \hat{u} \rrbracket_R =$$

$$= \bigcup_{v \in \mathrm{Dom}(u)} (\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket t = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \rangle \right\rrbracket_R \cap$$

$$\cap \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket t = r \rrbracket_R) \qquad (1)$$

$$t \in \mathrm{Sc}(\mathrm{rg}[v]), v \in \mathrm{Dom}(u) \vdash \neg\neg\, t \in \mathrm{rg}[u]$$

$$t \in \mathrm{Sc}(\mathrm{rg}[v]), v \in \mathrm{Dom}(u), t \notin \mathrm{rg}[u] \vdash$$

$$v \in \mathrm{Dom}(u) \vdash \neg \exists t \in \mathrm{Sc}(\mathrm{rg}[v])\, (t \notin \mathrm{rg}[u])$$

$$v \in \mathrm{Dom}(u) \vdash \mathrm{Sc}(\mathrm{rg}[v]) \subset' \mathrm{rg}[u];$$

$$\mathrm{Sc}(\mathrm{rg}[v]) \subset' \mathrm{rg}[u] \vdash$$

$$\vdash |R| \subset' \llbracket \check{\mathrm{Sc}}(\mathrm{rg}[v]) \subset' \mathrm{rg}[\check{u}] \rrbracket_R$$

$$v \in \mathrm{Dom}(u) \vdash |R| \subset' \llbracket \check{\mathrm{Sc}}(\mathrm{rg}[v]) \subset' \mathrm{rg}[\check{u}] \rrbracket_R;$$

$$V^C(S) \vdash \llbracket \check{\mathrm{Sc}}(\mathrm{rg}[v]) \subset' \mathrm{rg}[\check{u}] \rrbracket_R \subset'$$

$$\subset' \llbracket \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[v])}}_R \subset' \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R$$

$$v \in \mathrm{Dom}(u), V^C(S) \vdash |R| \subset' \llbracket \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[v])}}_R \subset' \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R$$

$$|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{v} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[v])}}_R \rrbracket_R,$$

$$v \in \mathrm{Dom}(u), V^C(S) \vdash |R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{v} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R$$

$$\forall r \in \mathrm{Dom}(u)\, (|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[r])}}_R \rrbracket_R),$$

$$v \in \mathrm{Dom}(u), V^C(S) \vdash |R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{v} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R \qquad (2)$$

$$
\frac{\begin{aligned}
V^C(\hat{v},t,y,S) \vdash {}& \llbracket \hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R \cap \llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset'_{\overline{R}\,\overline{R}} \\
&\subset'_{\overline{R}\,\overline{R}} \llbracket t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset' \\
&\subset'_{\overline{R}\,\overline{R}} \llbracket \underline{\langle t, y \rangle}_R \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S) \rrbracket_R \subset' \\
&\subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R
\end{aligned}}{\begin{aligned}
V^C(\hat{v},t,y,S) \vdash {}& \llbracket \hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R \cap \llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset' \\
&\subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R
\end{aligned}}
$$

$$
\frac{\begin{aligned}
(2) \Longrightarrow\ & \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[\check{r}])}}_R \rrbracket_R), \\
& v \in \mathrm{Dom}(u)\ V^C(S) \vdash |R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}[\check{u}]}}_R \rrbracket_R\,;
\end{aligned}}{\begin{aligned}
&\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[\check{r}])}}_R \rrbracket_R), V^C(\hat{v},t,y,S), \\
&v \in \mathrm{Dom}(u) \vdash \llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R \qquad (3)
\end{aligned}}
$$

$$
\frac{\dfrac{\begin{aligned}
& v \in \mathrm{Dom}(u), V^D(u) \vdash V^D(v)\,; \\
& \qquad V^D(v) \vdash V^C(\hat{v})
\end{aligned}}{v \in \mathrm{Dom}(u), V^D(u) \vdash V^C(\hat{v})\,; \qquad (3)}}{\dfrac{\begin{aligned}
&\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[\check{r}])}}_R \rrbracket_R), V^C(t,y,S), V^D(u), \\
&v \in \mathrm{Dom}(u) \vdash \llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R
\end{aligned}}{\begin{aligned}
&\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[\check{r}])}}_R \rrbracket_R), V^C(t,y,S), V^D(u) \vdash \\
&\vdash \forall v \in \mathrm{Dom}(u)(\,\llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \subset'_{\overline{R}\,\overline{R}} \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R) \ (4)
\end{aligned}}}
$$

$$
\begin{aligned}
(1) \underset{\langle t, y\rangle_R}{\overset{t}{\Longrightarrow}}\ & V^C(\underline{\langle t, y \rangle}_R), V^D(u), V^C(S) \vdash \llbracket \underline{\langle t, y \rangle}_R \in \hat{u} \rrbracket_R = \\
&= \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \left\llbracket \underline{\langle t, y \rangle}_R = \langle\, \hat{v}, \mathrm{S\mathring{h}}(\,\llbracket v \in u \rrbracket_Q, R, S)\,\rangle \right\rrbracket_R \cap \\
&\qquad \cap \bigcup_{r \in \mathrm{Dom}(\hat{u})} \llbracket \underline{\langle t, y \rangle}_R = r \rrbracket_R\,) \qquad (5)
\end{aligned}
$$

Схема 2.4.3

$$\underline{V^{C}(t,y) \vdash V^{C}(\langle t, y\rangle_{R});} \qquad (5)$$

$$V^{C}(t,y), V^{D}(u), V^{C}(S) \vdash [\![\underline{\langle t, y\rangle}_{R} \in \hat{u}]\!]_{R} =$$
$$= \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap \Big[\!\!\Big[\underline{\langle t, y\rangle}_{R} = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S)\,\rangle \Big]\!\!\Big]_{R} \cap$$
$$\cap \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,)$$

$$V^{C}(t,y), V^{D}(u), V^{C}(S) \vdash [\![\underline{\langle t, y\rangle}_{R} \in \hat{u}]\!]_{R} =$$
$$= \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap \Big[\!\!\Big[\underline{\langle t, y\rangle}_{R} = \underline{\langle \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S)\,\rangle}_{R} \Big]\!\!\Big]_{R} \cap$$
$$\cap \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,)$$

$$V^{C}(t,y,S), V^{D}(u) \vdash \overline{}_{R}\,\overline{}_{R}[\![\underline{\langle t, y\rangle}_{R} \in \hat{u}]\!]_{R} =$$
$$= \overline{}_{R}\,\overline{}_{R} \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap \Big[\!\!\Big[\underline{\langle t, y\rangle}_{R} = \underline{\langle \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S)\,\rangle}_{R} \Big]\!\!\Big]_{R} \cap$$
$$\cap \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,) =$$
$$= \overline{}_{R}\,\overline{}_{R} \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap [\![t = \hat{v}]\!]_{R} \cap$$
$$\cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S) \Big]\!\!\Big]_{R} \cap$$
$$\cap \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,) =$$
$$= \overline{}_{R}\,\overline{}_{R} \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap [\![t = \hat{v}]\!]_{R} \cap [\![y \in \underline{\mathrm{O}}_{R}(S)]\!]_{R} \cap$$
$$\cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S) \Big]\!\!\Big]_{R} \cap$$
$$\cap \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,) =$$
$$= \overline{}_{R}\,\overline{}_{R} \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap [\![t = \hat{v}]\!]_{R} \cap [\![y \in \underline{\mathrm{O}}_{R}(S)]\!]_{R} \cap$$
$$\cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S) \Big]\!\!\Big]_{R} \cap$$
$$\cap \overline{}_{R}\,\overline{}_{R} \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,)$$

$$V^{C}(t,y,S), V^{D}(u) \vdash \overline{}_{R}\,\overline{}_{R}[\![\underline{\langle t, y\rangle}_{R} \in \hat{u}]\!]_{R} =$$
$$= \overline{}_{R}\,\overline{}_{R} \bigcup_{v\in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v\in u]\!]_{Q}) \cap [\![t = \hat{v}]\!]_{R} \cap [\![y \in \underline{\mathrm{O}}_{R}(S)]\!]_{R} \cap$$
$$\cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v\in u]\!]_{Q}, R, S) \Big]\!\!\Big]_{R} \cap$$
$$\cap \overline{}_{R}\,\overline{}_{R} \bigcup_{r\in \mathrm{Dom}(\hat{u})} [\![\underline{\langle t, y\rangle}_{R} = r]\!]_{R}\,) \qquad (6)$$

$$
\frac{(6)\,; \qquad (4)}{\begin{array}{l}
V^{C}(t,y,S), V^{D}(u), \\
\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])_R}} \rrbracket_R) \vdash \\
\vdash \overline{}_R\,\overline{}_R \llbracket \underline{\langle t, y \rangle}_R \in \hat{u} \rrbracket_R = \\
= \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \llbracket t = \hat{v} \rrbracket_R \cap \llbracket y \in \underline{\mathrm{O}}_R(S) \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket y = \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \right\rrbracket_R) = \\
= \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \llbracket t = \hat{v} \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket y = \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \right\rrbracket_R) =
\end{array}}
$$

$$
\begin{array}{l}
V^{C}(t,y,S), V^{D}(u), \\
\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R \llbracket \hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])_R}} \rrbracket_R) \vdash \\
\vdash \overline{}_R\,\overline{}_R \llbracket \underline{\langle t, y \rangle}_R \in \hat{u} \rrbracket_R = \\
= \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \llbracket t = \hat{v} \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket y = \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \right\rrbracket_R)
\end{array} \qquad (7)
$$

$$
\frac{\begin{array}{l}
V^{C}(\mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S)), V^{C}(\hat{t}), t \in \mathrm{Dom}(u) \vdash \\
\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_Q) \subset' \mathrm{Dom}(\llbracket t \in u \rrbracket_Q) \cap |R| = \\
\qquad\qquad = \mathrm{Dom}(\llbracket t \in u \rrbracket_Q) \cap \llbracket \hat{t} = \hat{t} \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket \mathring{\mathrm{Sh}}(\llbracket t \in u \rrbracket_Q, R, S) = \mathring{\mathrm{Sh}}(\llbracket t \in u \rrbracket_Q, R, S) \right\rrbracket_R \subset' \\
\qquad\qquad \subset' \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \llbracket \hat{t} = \hat{v} \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket \mathring{\mathrm{Sh}}(\llbracket t \in u \rrbracket_Q, R, S) = \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \right\rrbracket_R)
\end{array}}{\begin{array}{l}
V^{C}(\mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S)), V^{C}(\hat{t}), t \in \mathrm{Dom}(u) \vdash \\
\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_Q) \subset' \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_Q) \cap \llbracket \hat{t} = \hat{v} \rrbracket_R \cap \\
\qquad\qquad \cap \left\llbracket \mathring{\mathrm{Sh}}(\llbracket t \in u \rrbracket_Q, R, S) = \mathring{\mathrm{Sh}}(\llbracket v \in u \rrbracket_Q, R, S) \right\rrbracket_R)
\end{array}} \qquad (8)
$$

$$(8) \Rightarrow\ V^{C}(\mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S)),\ V^{C}(\hat{t}),\ t \in \mathrm{Dom}(u) \vdash$$
$$\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_{Q}) \subset' \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_{Q}) \cap \llbracket \hat{t} = \hat{v} \rrbracket_{R} \cap$$
$$\cap \left\llbracket \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) = \mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S) \right\rrbracket_{R});$$

$$V^{C}(\hat{t}, \mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S), S),\ V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{O_R(S)}_{\breve{\mathrm{Sc}}(\mathrm{rg}[r])}}_{R} \rrbracket_{R}) \vdash$$
$$\vdash_{\overline{R}\,\overline{R}} \left\llbracket \underline{\langle \hat{t}, \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle}_{R} \in \hat{u} \right\rrbracket_{R} =$$
$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}(\llbracket v \in u \rrbracket_{Q}) \cap \llbracket t = \hat{v} \rrbracket_{R} \cap$$
$$\cap \left\llbracket \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) = \mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S) \right\rrbracket_{R}) \quad \overset{t \quad y}{\underset{\hat{t} \quad \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S)}{\Longleftarrow}} \quad (7)$$

$$V^{C}(\mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S)),\ V^{C}(\hat{t}),\ t \in \mathrm{Dom}(u),\ V^{C}(S),\ V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{O_R(S)}_{\breve{\mathrm{Sc}}(\mathrm{rg}[r])}}_{R} \rrbracket_{R}) \vdash$$
$$\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_{Q}) \subset'_{\overline{R}\,\overline{R}} \left\llbracket \underline{\langle \hat{t}, \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle}_{R} \in \hat{u} \right\rrbracket_{R} =$$
$$=_{\overline{R}\,\overline{R}} \left\llbracket \langle \hat{t}, \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \in \hat{u} \right\rrbracket_{R};$$

$$V^{C}(\mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S)),\ V^{C}(\hat{t}),\ V^{C}(\hat{u}) \vdash$$
$$\vdash_{\overline{R}\,\overline{R}} \left\llbracket \langle \hat{t}, \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \in \hat{u} \right\rrbracket_{R} \cap \llbracket \mathrm{Fn}(\hat{u}) \rrbracket_{R} \subset'$$
$$\subset' \left\llbracket \hat{u}(\hat{t}) = \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \right\rrbracket_{R}$$

$$V^{C}(\mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S)),\ V^{C}(\hat{t}),\ t \in \mathrm{Dom}(u),\ V^{C}(S),\ V^{D}(u),\ V^{C}(\hat{u}) \vdash$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{O_R(S)}_{\breve{\mathrm{Sc}}(\mathrm{rg}[r])}}_{R} \rrbracket_{R}) \vdash$$
$$\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_{Q}) \cap \llbracket \mathrm{Fn}(\hat{u}) \rrbracket_{R} \subset' \left\llbracket \hat{u}(\hat{t}) = \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \right\rrbracket_{R}$$
$$V^{C}(S) \vdash V^{C}(\mathring{\breve{\mathrm{Sh}}}(\llbracket v \in u \rrbracket_{Q}, R, S));$$

$$V^{C}(S),\ V^{C}(\hat{t}),\ t \in \mathrm{Dom}(u),\ V^{D}(u),\ V^{C}(\hat{u}) \vdash$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{O_R(S)}_{\breve{\mathrm{Sc}}(\mathrm{rg}[r])}}_{R} \rrbracket_{R}) \vdash$$
$$\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_{Q}) \cap \llbracket \mathrm{Fn}(\hat{u}) \rrbracket_{R} \subset' \left\llbracket \hat{u}(\hat{t}) = \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \right\rrbracket_{R}$$

$$|R| \subset' \llbracket \mathrm{Fn}(\hat{u}) \rrbracket_{R},\ V^{C}(S),\ V^{C}(\hat{t}),\ t \in \mathrm{Dom}(u),\ V^{D}(u),\ V^{C}(\hat{u}) \vdash$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} \llbracket \hat{r} \in \underline{\mathrm{V}^{O_R(S)}_{\breve{\mathrm{Sc}}(\mathrm{rg}[r])}}_{R} \rrbracket_{R}) \vdash$$
$$\vdash \mathrm{Dom}(\llbracket t \in u \rrbracket_{Q}) \subset' \left\llbracket \hat{u}(\hat{t}) = \mathring{\breve{\mathrm{Sh}}}(\llbracket t \in u \rrbracket_{Q}, R, S) \rangle \right\rrbracket_{R} \qquad (9)$$

Схема 2.4.6

(7)

$$V^{C}(t,S), V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash$$
$$\vdash \overline{}_R\,\overline{}_R [\![t \in \underline{\mathrm{Dom}}_R(\hat{u})]\!]_R = \overline{}_R\,\overline{}_R \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![t = \hat{v}]\!]_R\,)$$

$$V^{C}(t,S), V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash$$
$$\vdash \overline{}_Q\,\overline{}_Q [\![\mathrm{h}t \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\hat{u})]\!]_Q = \overline{}_Q\,\overline{}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}t = \mathrm{h}\hat{v}]\!]_R\,)$$

$$V^{C}(S), V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash \forall t\,(\,V^{C}(\,t\,) \rightarrow$$
$$\rightarrow \overline{}_Q\,\overline{}_Q [\![\mathrm{h}t \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\hat{u})]\!]_Q = \overline{}_Q\,\overline{}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}t = \mathrm{h}\hat{v}]\!]_R\,))$$

$$V^{C}(S), V^{D}(u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash \forall r\,(\,V^{C}(\,r\,) \rightarrow$$
$$\rightarrow \overline{}_Q\,\overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\hat{u})]\!]_Q = \overline{}_Q\,\overline{}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R\,))\,;$$

$$V^{D}(t,u) \vdash$$
$$\vdash \overline{}_Q\,\overline{}_Q [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$
$$= \overline{}_Q\,\overline{}_Q \bigcup_{V^{C}(r)} (\overline{}_Q\,\overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\hat{u})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(r)}_R \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$$

$$V^{C}(S), V^{D}(t,u),$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{}_R\,\overline{}_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash$$
$$\vdash \overline{}_Q\,\overline{}_Q [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$
$$= \overline{}_Q\,\overline{}_Q \bigcup_{V^{C}(r)} (\,\overline{}_Q\,\overline{}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R\,) \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(r)}_R \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

Схема 2.4.7

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \Big(\bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R\,) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(r)}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \Big) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(r)}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} \bigcup_{V^C(r)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap \bigcup_{v \in V^C(r)} [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap \bigcup_{v \in V^C(r)} [\![\mathrm{h}r = \mathrm{h}\hat{v}]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$=_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$V^C(S), V^D(t,u),$

$\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash$ $\Longrightarrow$

Схема 2.4.8

$$
\begin{aligned}
\Longrightarrow\ & \vdash_{\overline{Q}\,\overline{Q}} [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q = \\
& =_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\ \mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)
\end{aligned}
$$

$$
\begin{aligned}
& V^C(S), V^D(t,u), \\
& \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])\,R}}]\!]_R), \\
& \forall v \in \mathrm{Dom}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_R) \vdash \\
& \vdash_{\overline{Q}\,\overline{Q}} [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q = \\
& =_{\overline{Q}\,\overline{Q}} \bigcup_{v \in \mathrm{Dom}(u)} (\ \mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap \\
& \qquad \cap [\![t = v]\!]_Q)
\end{aligned} \qquad (10)
$$

$$
\begin{aligned}
& |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S), V^C(\hat{v}), v \in \mathrm{Dom}(u), V^D(u), V^C(\hat{u}) \vdash \\
& \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])\,R}}]\!]_R) \vdash \\
& \vdash \mathrm{Dom}([\![v \in u]\!]_Q) \subset' \left[\!\!\left[\hat{u}(\hat{v}) = \mathrm{Sh}\mathring{(}\, [\![v \in u]\!]_Q, R, S) \rangle \right]\!\!\right]_R \quad \overset{t}{\underset{v}{\Longleftarrow}} (9)
\end{aligned}
$$

$$
\begin{aligned}
& |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S), V^C(\hat{v}), v \in \mathrm{Dom}(u), V^D(u), V^C(\hat{u}) \vdash \\
& \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])\,R}}]\!]_R) \vdash \\
& \vdash \mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \subset' \left[\!\!\left[\mathrm{h}\ \hat{u}(\hat{v}) = \mathrm{h}\ \mathrm{Sh}\mathring{(}\, [\![v \in u]\!]_Q, R, S) \rangle \right]\!\!\right]_Q
\end{aligned}
$$

$$
\begin{aligned}
& V^C(\hat{v})\ \&\ V^C(\hat{u}), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S), v \in \mathrm{Dom}(u), V^D(u) \vdash \\
& \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])\,R}}]\!]_R) \vdash \\
& \vdash \mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \subset' \left[\!\!\left[\mathrm{h}\ \hat{u}(\hat{v}) = \mathrm{h}\ \mathrm{Sh}\mathring{(}\, [\![v \in u]\!]_Q, R, S) \rangle \right]\!\!\right]_Q \\
& \qquad\qquad v \in \mathrm{Dom}(u), V^D(u) \vdash V^C(\hat{v})\ \&\ V^C(\hat{u});
\end{aligned}
$$

$$
\begin{aligned}
& \qquad\qquad v \in \mathrm{Dom}(u), V^D(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S), \\
& \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])\,R}}]\!]_R) \vdash \\
& \vdash \mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \subset' \left[\!\!\left[\mathrm{h}\ \hat{u}(\hat{v}) = \mathrm{h}\ \mathrm{Sh}\mathring{(}\, [\![v \in u]\!]_Q, R, S) \rangle \right]\!\!\right]_Q
\end{aligned} \qquad (11)
$$

С х е м а 2.4.9

$$\frac{(11)}{\begin{gathered}V^D(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R, V^C(S),\\ \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{\ }_R\overline{\ }_R[\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R) \vdash\\ \vdash \forall v \in \mathrm{Dom}(u)(\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \subset'\\ \subset' \Big[\!\!\Big[\mathrm{h}\,\hat{u}(\hat{v}) = \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\Big]\!\!\Big]_Q);\end{gathered}}$$

$$\frac{(10)}{\begin{gathered}V^C(S), V^D(t,u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R,\\ \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{\ }_R\overline{\ }_R[\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R),\\ \forall v \in \mathrm{Dom}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash\\ \vdash \overline{\ }_Q\overline{\ }_Q[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =\\ = \overline{\ }_Q\overline{\ }_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap\\ \cap \Big[\!\!\Big[\mathrm{h}\,\hat{u}(\hat{v}) = \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\Big]\!\!\Big]_Q \cap\\ \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{\hat{u}(\hat{v})}_R \neq 0]\!]_Q \cap\\ \cap [\![t = v]\!]_Q) =\\ = \overline{\ }_Q\overline{\ }_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap\\ \cap \Big[\!\!\Big[\mathrm{h}\,\hat{u}(\hat{v}) = \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\Big]\!\!\Big]_Q \cap\\ \cap \Big[\!\!\Big[\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle \neq 0\Big]\!\!\Big]_Q \cap\\ \cap [\![t = v]\!]_Q) =\\ = \overline{\ }_Q\overline{\ }_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap\\ \cap \Big[\!\!\Big[\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle \neq 0\Big]\!\!\Big]_Q \cap\\ \cap [\![t = v]\!]_Q) =\\ = \overline{\ }_Q\overline{\ }_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\overline{\ }_Q\overline{\ }_Q\mathrm{h}\,\mathrm{Dom}([\![v \in u]\!]_Q) \cap\\ \cap \Big[\!\!\Big[\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle \neq 0\Big]\!\!\Big]_Q \cap\\ \cap [\![t = v]\!]_Q)\end{gathered}}$$

$$\begin{gathered}V^C(S), V^D(t,u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R,\\ \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \overline{\ }_R\overline{\ }_R[\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{Sc}(\mathrm{rg}[r])}}_R]\!]_R),\end{gathered} \qquad \Longrightarrow$$

$$\Longrightarrow \quad \forall v \in \mathrm{Dom}(u)\,(\,|\,Q\,|\subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v},\, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash$$
$$\vdash \overline{\,}_Q\,\overline{\,}_Q[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u},\, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$
$$= \overline{\,}_Q\,\overline{\,}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\,\overline{\,}_Q\,\overline{\,}_Q\,\mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \cap$$
$$\cap \Big[\!\!\Big[\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \mathrm{Sh}\mathring{\langle}\,[\![v \in u]\!]_Q\,, R, S)\,\rangle \neq 0\Big]\!\!\Big]_Q \cap$$
$$\cap\,[\![t = v]\!]_Q)\,;$$
$$V^D(u) \vdash$$
$$\vdash \forall v \in \mathrm{Dom}(u)\,(\overline{\,}_Q\,\overline{\,}_Q\,\mathrm{h}\ \mathrm{Dom}([\![v \in u]\!]_Q) \cap$$
$$\cap \Big[\!\!\Big[\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \mathrm{Sh}\mathring{\langle}\,[\![v \in u]\!]_Q\,, R, S)\,\rangle \neq 0\Big]\!\!\Big]_Q =$$
$$= \overline{\,}_Q\,\overline{\,}_Q[\![v \in u]\!]_Q) \quad {}^{1)}$$

$$V^C(S), V^D(t,u), |\,R\,| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R\,,$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|\,R\,|\subset' \overline{\,}_R\,\overline{\,}_R[\![\hat{r} \in \underline{\mathrm{V}}^{\mathrm{O}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])\,R}]\!]_R),$$
$$\forall v \in \mathrm{Dom}(u)\,(\,|\,Q\,|\subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v},\, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash$$
$$\vdash \overline{\,}_Q\,\overline{\,}_Q[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u},\, \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$
$$= \overline{\,}_Q\,\overline{\,}_Q \bigcup_{v \in \mathrm{Dom}(u)} (\overline{\,}_Q\,\overline{\,}_Q[\![v \in u]\!]_Q \cap [\![t = v]\!]_Q) =$$
$$= \overline{\,}_Q\,\overline{\,}_Q \bigcup_{v \in \mathrm{Dom}(u)} ([\![v \in u]\!]_Q \cap [\![t = v]\!]_Q) =$$
$$= \overline{\,}_Q\,\overline{\,}_Q \bigcup_{v \in \mathrm{Dom}(u)} ([\![t \in u]\!]_Q \cap [\![t = v]\!]_Q) =$$
$$= \overline{\,}_Q\,\overline{\,}_Q(\,[\![t \in u]\!]_Q \cap \bigcup_{v \in \mathrm{Dom}(u)} [\![t = v]\!]_Q\,) =$$
$$= \overline{\,}_Q\,\overline{\,}_Q[\![t \in u]\!]_Q$$

$$V^C(S), V^D(t,u), |\,R\,| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R\,,$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|\,R\,|\subset' \overline{\,}_R\,\overline{\,}_R[\![\hat{r} \in \underline{\mathrm{V}}^{\mathrm{O}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])\,R}]\!]_R),$$
$$\forall v \in \mathrm{Dom}(u)\,(\,|\,Q\,|\subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v},\, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash$$
$$\vdash \overline{\,}_Q\,\overline{\,}_Q[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u},\, \underline{\mathrm{Rg}}_Q(G))]\!]_Q = \overline{\,}_Q\,\overline{\,}_Q[\![t \in u]\!]_Q$$

$$V^C(S), V^D(u), |\,R\,| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R\,,$$
$$\forall r \in \mathrm{Dom}(u)\,(\,|\,R\,|\subset' \overline{\,}_R\,\overline{\,}_R[\![\hat{r} \in \underline{\mathrm{V}}^{\mathrm{O}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])\,R}]\!]_R),$$
$$\forall v \in \mathrm{Dom}(u)\,(\,|\,Q\,|\subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v},\, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash$$
$$\vdash \forall t\,(\,V^D(\,t\,) \rightarrow \overline{\,}_Q\,\overline{\,}_Q[\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\hat{u},\, \underline{\mathrm{Rg}}_Q(G))]\!]_Q = \overline{\,}_Q\,\overline{\,}_Q[\![t \in u]\!]_Q\,) \qquad (12)$$

$$\begin{array}{l}
\qquad\qquad\qquad (12) \\ \hline
V^{C}(S), V^{D}(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R , \\
\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
\forall v \in \mathrm{Dom}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q) \vdash \\
\vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G)) = u]\!]_Q ; \\
\;|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{u}, \underline{\mathrm{Rg}}_Q(G)) = u]\!]_Q , u \in \breve{\mathrm{Dom}}(Z) \vdash \neg\neg\, u \in Y \\ \hline
V^{C}(S), V^{D}(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R , \\
\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
\forall v \in \mathrm{Dom}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) = v]\!]_Q), \\
u \in \breve{\mathrm{Dom}}(Z) \vdash \neg\neg\, u \in Y \\ \hline
V^{C}(S), V^{D}(u), |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R , \\
\forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
\mathrm{Dom}(u) \subset' Y, u \in \breve{\mathrm{Dom}}(Z) \vdash \neg\neg\, u \in Y \\ \hline
|R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R \;\&\; \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R) \\
V^{C}(S), V^{D}(u), \mathrm{Dom}(u) \subset' Y, u \in \breve{\mathrm{Dom}}(Z) \vdash \neg\neg\, u \in Y \qquad\qquad\qquad (13)
\end{array}$$

$$\begin{array}{l}
r \in \mathrm{Dom}(u), u \in \breve{\mathrm{Dom}}(Z) \vdash \neg\neg\, r \in \breve{\mathrm{Dom}}(Z) \\ \hline
\qquad u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R) \vdash \\
\qquad\qquad \vdash \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R) \\
\qquad u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
\qquad V^{C}(S) \vdash |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R ; \\ \hline
\qquad u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
V^{C}(S) \vdash |R| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R \;\&\; \forall r \in \mathrm{Dom}(u)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R) ; \\
\qquad\qquad\qquad\qquad (13) \\ \hline
\qquad u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset' \underset{R}{-}\,\underset{R}{-}[\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{Sc}\check{(}\mathrm{rg}[r])}}_{R}]\!]_R), \\
\qquad V^{C}(S), V^{D}(u), \mathrm{Dom}(u) \subset' Y \vdash \neg\neg\, u \in Y \qquad\qquad\qquad (14)
\end{array}$$

$$u \in \breve{\mathrm{Dom}}(Z), V^{D}(Z) \vdash V^{D}(u)\,;$$

$$\begin{array}{l}
u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
V^{C}(S), V^{D}(u), \mathrm{Dom}(u) \subset' Y \vdash \neg\neg\, u \in Y \qquad \Longleftarrow (14)\\
\hline
u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
V^{C}(S), V^{D}(Z), \mathrm{Dom}(u) \subset' Y \vdash \neg\neg\, u \in Y\\
\hline
u \in \breve{\mathrm{Dom}}(Z), \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
V^{C}(S), V^{D}(Z) \vdash \mathrm{Dom}(u) \subset' Y \to \neg\neg\, u \in Y\\
\hline
\qquad\qquad \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
V^{C}(S), V^{D}(Z) \vdash \forall u \in \breve{\mathrm{Dom}}(Z)\,(\mathrm{Dom}(u) \subset' Y \to \neg\neg\, u \in Y)\,;\\
\qquad\qquad\qquad\qquad \forall u \in \breve{\mathrm{Dom}}(Z)\,(\mathrm{Dom}(u) \subset' Y \to \neg\neg\, u \in Y)\\
\qquad\qquad\qquad\qquad V^{D}(Z) \vdash \breve{\mathrm{Dom}}(Z) \subset' Y\\
\hline
\qquad\qquad \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
\qquad\qquad V^{C}(S), V^{D}(Z) \vdash \breve{\mathrm{Dom}}(Z) \subset' Y\\
\hline
\qquad\qquad \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
\qquad\qquad V^{C}(S), V^{D}(Z) \vdash\\
\qquad\qquad \vdash \forall r \in \breve{\mathrm{Dom}}(Z)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q) \qquad (15)
\end{array}$$

$$\begin{array}{l}
\breve{\mathrm{Dom}}(u) \subset' X \vdash \forall r \in \breve{\mathrm{Dom}}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R)\,;\\
\qquad (15) \overset{Z}{\underset{u}{\Longrightarrow}} \forall r \in \breve{\mathrm{Dom}}(u)\,(\,|R| \subset'_{\overline{R}\,\overline{R}} [\![\hat{r} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\mathrm{S\check{c}}(\mathrm{rg}[r])_R}}]\!]_R),\\
\qquad\qquad V^{C}(S), V^{D}(u) \vdash\\
\qquad\qquad \vdash \forall r \in \breve{\mathrm{Dom}}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q)\\
\hline
\qquad\qquad \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S), V^{D}(u) \vdash\\
\qquad\qquad \vdash \forall r \in \breve{\mathrm{Dom}}(u)\,(\,|Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q)\\
\hline
\quad t \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S), V^{D}(u) \vdash\\
\qquad\qquad \vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{t}, \underline{\mathrm{Rg}}_Q(G)) = t]\!]_Q \qquad (16)
\end{array}$$

Схема 2.4.13

$$\frac{(16)}{\dfrac{t \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{t}, \underline{\mathrm{Rg}}_Q(G)) = t]\!]_Q}{t \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{t}, \underline{\mathrm{Rg}}_Q(G)) \in u]\!]_Q = [\![t \in u]\!]_Q}} \quad (17)$$

$$(17);$$

$$(17) \underset{v}{\overset{t}{\Longrightarrow}} v \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) \in u]\!]_Q = [\![v \in u]\!]_Q;$$

$$V^C(\hat{t}, \hat{v}), V^D(u), V^C(S) \vdash [\![\hat{t} = \hat{v}]\!]_R \subset' \subset' \left[\!\!\left[\mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{t}, \underline{\mathrm{Rg}}_Q(G)) \in u]\!]_Q, R, S) = \mathring{\mathrm{Sh}}([\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{v}, \underline{\mathrm{Rg}}_Q(G)) \in u]\!]_Q, R, S) \right]\!\!\right]_R \ ^{2)}$$

$$\overline{t, v \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u), V^C(\hat{t}, \hat{v}) \vdash \vdash [\![\hat{t} = \hat{v}]\!]_R \subset' \left[\!\!\left[\mathring{\mathrm{Sh}}([\![t \in u]\!]_Q, R, S) = \mathring{\mathrm{Sh}}([\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R}$$

$$V^C(\hat{t}, \hat{v}, r) \vdash [\![r = \hat{t}]\!]_R \cap [\![r = \hat{v}]\!]_R \subset' [\![\hat{t} = \hat{v}]\!]_R;$$

$$\overline{V^C(\hat{t}, \hat{v}, r),\ t, v \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash [\![r = \hat{t}]\!]_R \cap [\![r = \hat{v}]\!]_R \subset' \left[\!\!\left[\mathring{\mathrm{Sh}}([\![t \in u]\!]_Q, R, S) = \mathring{\mathrm{Sh}}([\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R}$$

$$\overline{V^C(\hat{t}, \hat{v}, r, y, z),\ t, v \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash [\![r = \hat{t}]\!]_R \cap [\![r = \hat{v}]\!]_R \cap \left[\!\!\left[y = \mathring{\mathrm{Sh}}([\![t \in u]\!]_Q, R, S) \right]\!\!\right]_R \cap \cap \left[\!\!\left[z = \mathring{\mathrm{Sh}}([\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R \subset' [\![y = z]\!]_R}$$

$$t, v \in \mathrm{Dom}(u), V^D(u) \vdash V^C(\hat{t}, \hat{v});$$

$$\overline{V^C(r, y, z),\ t, v \in \mathrm{Dom}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash [\![r = \hat{t}]\!]_R \cap [\![r = \hat{v}]\!]_R \cap \left[\!\!\left[y = \mathring{\mathrm{Sh}}([\![t \in u]\!]_Q, R, S) \right]\!\!\right]_R \cap \cap \left[\!\!\left[z = \mathring{\mathrm{Sh}}([\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R \subset' [\![y = z]\!]_R}$$

$$\overline{V^C(r, y, z), \breve{\mathrm{Dom}}(u) \subset' X, V^C(S), V^D(u) \vdash \vdash \forall t, v \in \mathrm{Dom}(u) \left([\![r = \hat{t}]\!]_R \cap [\![r = \hat{v}]\!]_R \cap \left[\!\!\left[y = \mathring{\mathrm{Sh}}([\![t \in u]\!]_Q, R, S) \right]\!\!\right]_R \cap \cap \left[\!\!\left[z = \mathring{\mathrm{Sh}}([\![v \in u]\!]_Q, R, S) \right]\!\!\right]_R \subset' [\![y = z]\!]_R \right)} \quad (18)$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad(18)\\
\hline
\quad V^{C}(r,y,z), \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S), V^{D}(u) \vdash \\
\vdash \forall t, v \in \mathrm{Dom}(u)\,(\,[\![r = \hat{t}]\!]_R \cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![t \in u]\!]_Q, R, S) \Big]\!\!\Big]_R \cap \\
\qquad\qquad \cap [\![r = \hat{v}]\!]_R \cap \Big[\!\!\Big[z = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \Big]\!\!\Big]_R \subset' [\![y = z]\!]_R\,) \\
\hline
\quad V^{C}(r,y,z), \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S), V^{D}(u) \vdash \\
\vdash \bigcup\limits_{t \in \mathrm{Dom}(u)} (\,[\![r = \hat{t}]\!]_R \cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![t \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,) \cap \\
\cap \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,[\![r = \hat{v}]\!]_R \cap \Big[\!\!\Big[z = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,) \subset' [\![y = z]\!]_R \\
\hline
\quad V^{C}(r,y,z), \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S), V^{D}(u) \vdash \\
\vdash \overline{}_R\,\overline{}_R \bigcup\limits_{t \in \mathrm{Dom}(u)} (\,[\![r = \hat{t}]\!]_R \cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![t \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,) \cap \\
\cap \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,[\![r = \hat{v}]\!]_R \cap \Big[\!\!\Big[z = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,) \subset' [\![y = z]\!]_R
\end{array}
\qquad (19)
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad(6)\\
\hline
V^{C}(r,y,S), V^{D}(u) \vdash \overline{}_R\,\overline{}_R [\![\underline{\langle r, y \rangle}_R \in \hat{u}]\!]_R \subset' \\
\subset' \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,[\![r = \hat{v}]\!]_R \cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,) \\
V^{C}(r,y), V^{D}(u) \vdash [\![\langle r, y \rangle \in \hat{u}]\!]_R \subset' \overline{}_R\,\overline{}_R [\![\underline{\langle r, y \rangle}_R \in \hat{u}]\!]_R\,; \\
\hline
V^{C}(r,y,S), V^{D}(u) \vdash [\![\langle r, y \rangle \in \hat{u}]\!]_R \subset' \\
\subset' \overline{}_R\,\overline{}_R \bigcup\limits_{v \in \mathrm{Dom}(u)} (\,[\![r = \hat{v}]\!]_R \cap \Big[\!\!\Big[y = \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,)
\end{array}
\qquad (20)
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad(20)\,; \\
(20) \overset{y}{\underset{z}{\Longrightarrow}} \; V^{C}(r,z,S), V^{D}(u) \vdash [\![\langle r, z \rangle \in \hat{u}]\!]_R \subset' \\
\subset' \overline{}_R\,\overline{}_R \bigcup\limits_{t \in \mathrm{Dom}(u)} (\,[\![r = \hat{t}]\!]_R \cap \Big[\!\!\Big[z = \mathrm{Sh}\mathring{(}\,[\![t \in u]\!]_Q, R, S) \Big]\!\!\Big]_R\,)\,; \\
\qquad\qquad\qquad\qquad(19) \\
\hline
V^{C}(r,y,z,S), V^{D}(u), \breve{\mathrm{Dom}}(u) \subset' X, V^{C}(S) \vdash \\
\vdash [\![\langle r, y \rangle \in \hat{u}]\!]_R \cap [\![\langle r, z \rangle \in \hat{u}]\!]_R \subset' [\![y = z]\!]_R
\end{array}
\qquad (21)
$$

$$(21)$$

$$V^C(r,y,z,S), V^D(u), \breve{\mathrm{Dom}}(u) \subset' X \vdash$$
$$\vdash |R| \subset' (\llbracket \langle r, y\rangle \in \hat{u}\rrbracket_R \cap \llbracket \langle r, z\rangle \in \hat{u}\rrbracket_R \underset{R}{\supset} \llbracket y = z\rrbracket_R)$$

$$V^C(r,y,z,S), V^D(u), \breve{\mathrm{Dom}}(u) \subset' X \vdash$$
$$\vdash |R| \subset' \llbracket \langle r, y\rangle \in \hat{u} \;\&\; \langle r, z\rangle \in \hat{u} \to y = z\rrbracket_R$$

$$V^C(S), V^D(u), \breve{\mathrm{Dom}}(u) \subset' X \vdash$$
$$\vdash \forall r,y,z\,(V^C(r,y,z) \to |R| \subset' \llbracket \langle r, y\rangle \in \hat{u} \;\&\; \langle r, z\rangle \in \hat{u} \to y = z\rrbracket_R)$$

$$V^C(S), V^D(u), \breve{\mathrm{Dom}}(u) \subset' X \vdash$$
$$\vdash |R| \subset' \llbracket \forall r,y,z\,(\langle r, y\rangle \in \hat{u} \;\&\; \langle r, z\rangle \in \hat{u} \to y = z)\rrbracket_R \qquad (22)$$

$$(1)$$

$$V^C(t), V^D(u), V^C(S) \vdash \llbracket t \in \hat{u}\rrbracket_R \subset'$$
$$\subset' \bigcup_{v \in \mathrm{Dom}(u)} \llbracket t = \langle \hat{v}, \mathring{\mathrm{Sh}}(\llbracket v \in u\rrbracket_Q, R, S)\rangle\rrbracket_R \subset' \bigcup_{V^C(y)} \bigcup_{V^C(z)} \llbracket t = \langle y, z\rangle\rrbracket_R =$$
$$= \llbracket \exists y,z\,(t = \langle y, z\rangle)\rrbracket_R \subset' \underset{R}{\neg}\underset{R}{\neg}\llbracket \exists y,z\,(t = \langle y, z\rangle)\rrbracket_R =$$
$$= \llbracket \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R$$

$$V^C(t), V^D(u), V^C(S) \vdash \llbracket t \in \hat{u}\rrbracket_R \subset' \llbracket \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R$$

$$V^C(t), V^D(u), V^C(S) \vdash |R| \subset' (\llbracket t \in \hat{u}\rrbracket_R \underset{R}{\supset} \llbracket \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R)$$

$$V^C(t), V^D(u), V^C(S) \vdash |R| \subset' \llbracket t \in \hat{u} \to \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R$$

$$V^D(u), V^C(S) \vdash \forall t\,(V^C(t) \to$$
$$\to |R| \subset' \llbracket t \in \hat{u} \to \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R)$$

$$V^D(u), V^C(S) \vdash |R| \subset' \llbracket \forall t \in \hat{u}\;\; \neg\neg\exists y,z\,(t = \langle y, z\rangle)\rrbracket_R$$

$$V^D(u), V^C(S) \vdash |R| \subset' \llbracket \mathrm{Rl}(\hat{u})\rrbracket_R; \qquad (22)$$

$$V^D(u), V^C(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash$$
$$\vdash |R| \subset' \llbracket \mathrm{Rl}(\hat{u})\rrbracket_R \cap \llbracket \forall r,y,z\,(\langle r, y\rangle \in \hat{u} \;\&\; \langle r, z\rangle \in \hat{u} \to y = z)\rrbracket_R =$$
$$\subset' \llbracket \mathrm{Rl}(\hat{u}) \;\&\; \forall r,y,z\,(\langle r, y\rangle \in \hat{u} \;\&\; \langle r, z\rangle \in \hat{u} \to y = z)\rrbracket_R =$$
$$= \llbracket \mathrm{Fn}(\hat{u})\rrbracket_R$$

$$V^D(u), V^C(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash |R| \subset' \llbracket \mathrm{Fn}(\hat{u})\rrbracket_R \qquad (23)$$

$$
\frac{(2)}{}
$$

$$
v \in \mathrm{Dom}(u), \mathrm{Dom}(u) \subset' X, V^{C}(S) \vdash |R| \subset' \overline{\ }_{R}\overline{\ }_{R}[\![\hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R}]\!]_R
$$

$$
V^{C}(S) \vdash |R| \subset' \overline{\ }_{R}\overline{\ }_{R}\left[\!\!\left[\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \in \underline{\mathrm{O}}_R(S)\right]\!\!\right]_R ;
$$

$$
\begin{aligned}
V^{C}(\hat{v}), V^{C}(S), V^{C}(t) \vdash \overline{\ }_{R}\overline{\ }_{R}[\![\hat{v} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R}]\!]_R \cap \\
\cap \overline{\ }_{R}\overline{\ }_{R}\left[\!\!\left[\mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S) \in \underline{\mathrm{O}}_R(S)\right]\!\!\right]_R \cap \\
\cap \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R \subset' \\
\subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R
\end{aligned}
$$

$$
\begin{aligned}
v \in \mathrm{Dom}(u), \mathrm{Dom}(u) \subset' X, V^{C}(S), V^{C}(\hat{v}), V^{C}(t) \vdash \\
\vdash \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R \subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R
\end{aligned}
$$

$$
v \in \mathrm{Dom}(u), V^{D}(u) \vdash V^{C}(\hat{v}) ;
$$

$$
\begin{aligned}
v \in \mathrm{Dom}(u), \mathrm{Dom}(u) \subset' X, V^{C}(S), V^{C}(t), V^{D}(u) \vdash \\
\vdash \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R \subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Dom}(u) \subset' X, V^{C}(S), V^{C}(t), V^{D}(u) \vdash \forall v (\, v \in \mathrm{Dom}(u) \rightarrow \\
\rightarrow \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R \subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R\,)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Dom}(u) \subset' X, V^{C}(S), V^{C}(t), V^{D}(u) \vdash \\
\vdash \bigcup_{v \in \mathrm{Dom}(u)} \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R \subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R
\end{aligned}
$$

$$
\begin{aligned}
(1) \Longrightarrow \quad V^{C}(t), V^{D}(u), V^{C}(S) \vdash [\![t \in \hat{u}]\!]_R \subset' \\
\subset' \bigcup_{v \in \mathrm{Dom}(u)} \left[\!\!\left[t = \langle\, \hat{v}, \mathrm{Sh}\mathring{(}\,[\![v \in u]\!]_Q, R, S)\,\rangle\right]\!\!\right]_R ;
\end{aligned}
$$

$$
\begin{aligned}
V^{C}(t), V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\
\vdash [\![t \in \hat{u}]\!]_R \subset' \overline{\ }_{R}\overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R
\end{aligned}
$$

$$
\begin{aligned}
V^{C}(t), V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\
\vdash [\![t \in \hat{u}]\!]_R \cap \overline{\ }_{R}[\![t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R \subset' 0
\end{aligned}
$$

$$
\begin{aligned}
V^{C}(t), V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\
\vdash [\![t \in \hat{u} \;\&\; \neg\, t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R \subset' 0
\end{aligned}
$$

$$
\begin{aligned}
V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \forall t\, (V^{C}(t) \rightarrow \\
\rightarrow [\![t \in \hat{u} \;\&\; \neg\, t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R \subset' 0)
\end{aligned}
$$

$$
\begin{aligned}
V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\
\vdash \bigcup_{V^{C}(t)} [\![t \in \hat{u} \;\&\; \neg\, t \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\mathrm{rg}\check{[u]}}}_{R} \underline{\times}_R \underline{\mathrm{O}}_R(S)]\!]_R \subset' 0 \qquad (24)
\end{aligned}
$$

$$\frac{(24)}{\begin{array}{l} V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\ \vdash [\![\exists t \in \hat{u}\,(\neg\, t \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \subset' 0 \end{array}}$$

$$\begin{array}{l} V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\ \vdash |\, R \,| \subset' \overline{}_R [\![\exists t \in \hat{u}\,(\neg\, t \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R = \\ \qquad = [\![\neg \exists t \in \hat{u}\,(\neg\, t \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R = \\ \qquad = [\![\hat{u} \subset' \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \end{array}$$

$$\begin{array}{l} V^{D}(u), V^{C}(S), \mathrm{Dom}(u) \subset' X \vdash \\ \vdash |\, R \,| \subset' [\![\hat{u} \subset' \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \end{array}$$

$$(23) \Longrightarrow V^{D}(u), V^{C}(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash |\, R \,| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R ;$$

$$\begin{array}{l} V^{D}(u), V^{C}(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash \\ \vdash |\, R \,| \subset' [\![\mathrm{Fn}(\hat{u})]\!]_R \cap [\![\hat{u} \subset' \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R = \\ \qquad = \overline{}_R \overline{}_R [\![\hat{u} \in \underline{\mathrm{PF}}_R (\underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{rg}}[u]}}_R \underline{\times}_R \underline{\mathrm{O}}_R(S)))]\!]_R = \\ \qquad = \overline{}_R \overline{}_R [\![\hat{u} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[u])}}_R]\!]_R \end{array}$$

$$\begin{array}{l} V^{D}(u), V^{C}(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash \\ \vdash |\, R \,| \subset' \overline{}_R \overline{}_R [\![\hat{u} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[u])}}_R]\!]_R ; \\ |\, R \,| \subset' \overline{}_R \overline{}_R [\![\hat{u} \in \underline{\mathrm{V}^{\underline{\mathrm{O}}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[u])}}_R]\!]_R , u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\} \vdash \neg\neg\, u \in X \end{array}$$

$$\begin{array}{l} V^{D}(u), V^{C}(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash \\ u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\} \vdash \neg\neg\, u \in X \\ u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\}, V^{D}(U) \vdash V^{D}(u) ; \end{array}$$

$$\begin{array}{l} u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\}, V^{D}(U), V^{C}(S), \breve{\mathrm{Dom}}(u) \subset' X \vdash \\ \vdash \neg\neg\, u \in X \end{array}$$

$$\begin{array}{l} \qquad\qquad V^{D}(U), V^{C}(S) \vdash \\ \vdash \forall u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\}\,(\, \breve{\mathrm{Dom}}(u) \subset' X \to \neg\neg\, u \in X\,) ; \\ \forall u \in \mathrm{TC}_{\prec}(\{U\}) \cup \{U\}\,(\, \breve{\mathrm{Dom}}(u) \subset' X \to \neg\neg\, u \in X\,) \vdash \\ \vdash \mathrm{TC}_{\prec}(\{U\}) \cup \{U\} \subset' X \end{array}$$

$$V^{D}(U), V^{C}(S) \vdash \mathrm{TC}_{\prec}(\{U\}) \cup \{U\} \subset' X \qquad (25)$$

$$V^{D}(G), V^{C}(u,v,S) \vdash$$

$$\vdash [\![G \cap \mathrm{h}\, \underline{[\![u \in v]\!]_{SR}} \neq 0]\!]_{Q} =$$

$$= [\![G \cap \mathrm{h} \underline{\bigcup \underline{\mathrm{Ev}_{=R}}(u,v,S)_{R}} \neq 0]\!]_{Q} =$$

$$= [\![\neg\neg\exists t \in \mathrm{h}\, \underline{\mathrm{Ev}_{=R}}(u,v,S)(G \cap t \neq 0)]\!]_{Q} \cap$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{D}(t)} (\overline{\ }_{Q}\, \overline{\ }_{Q} [\![t \in \mathrm{h}\, \underline{\mathrm{Ev}_{=R}}(u,v,S)]\!]_{Q} \cap$$

$$\cap [\![G \cap t \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{D}(t)} (\overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q}) \cap$$

$$\cap [\![G \cap t \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{D}(t)} (\bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q}) \cap$$

$$\cap [\![G \cap t \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{D}(t)} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q}) \cap$$

$$\cap [\![G \cap t \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{C}(r)} \bigcup_{V^{D}(t)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q}) \cap$$

$$\cap [\![G \cap t \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{C}(r)} \bigcup_{V^{D}(t)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q}) \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}} \neq 0]\!]_{Q}) =$$

$$= \overline{\ }_{Q}\, \overline{\ }_{Q} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap$$

$$\cap \bigcup_{V^{D}(t)} [\![t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}}]\!]_{Q} \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{[\![u = r]\!]_{SR}} \neq 0]\!]_{Q}) =$$

Схема 2.5.2

$$
\begin{array}{l}
= {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{R}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q}) = \\
= {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{Q}\mathrm{h}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q}) \\
\hline
V^{D}(G), V^{C}(u,v,S) \vdash \\
\vdash \llbracket G \cap \mathrm{h}\, \underline{\llbracket u \in v\rrbracket}_{SR} \neq 0\rrbracket_{Q} = \\
= {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{Q}\mathrm{h}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q})
\end{array}
\tag{1}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad (1) \\
\hline
V^{D}(G), V^{C}(u,v,S) \vdash \\
\vdash \llbracket \mathrm{Fn}(\mathrm{h}v)\ \&\ \mathrm{h}\,\underline{\mathrm{Rg}}_{R}(v) \subset' \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket u \in v\rrbracket}_{SR} \neq 0\rrbracket_{Q} = \\
= \llbracket \mathrm{Fn}(\mathrm{h}v)\ \&\ \mathrm{h}\,\underline{\mathrm{Rg}}_{R}(v) \subset' \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\cap {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{Q}\mathrm{h}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q}) = \\
= \llbracket \mathrm{Fn}(\mathrm{h}v)\ \&\ \mathrm{h}\,\underline{\mathrm{Rg}}_{R}(v) \subset' \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\cap {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap {}_{\overline{Q}\,\overline{Q}}\llbracket \mathrm{h}\,\underline{v(r)}_{R} \in \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{Q}\mathrm{h}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q}) \\
\hline
V^{D}(G), V^{C}(u,v,S) \vdash \\
\vdash \llbracket \mathrm{Fn}(\mathrm{h}v)\ \&\ \mathrm{h}\,\underline{\mathrm{Rg}}_{R}(v) \subset' \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket u \in v\rrbracket}_{SR} \neq 0\rrbracket_{Q} = \\
= \llbracket \mathrm{Fn}(\mathrm{h}v)\ \&\ \mathrm{h}\,\underline{\mathrm{Rg}}_{R}(v) \subset' \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\cap {}_{\overline{Q}\,\overline{Q}}\bigcup\limits_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)\rrbracket_{Q} \cap \\
\qquad \cap {}_{\overline{Q}\,\overline{Q}}\llbracket \mathrm{h}\,\underline{v(r)}_{R} \in \mathrm{h}\,\underline{\mathrm{O}}_{R}(S)\rrbracket_{Q} \cap \\
\qquad \cap \llbracket G \cap \mathrm{h}\,\underline{v(r)}_{R} \,\underline{\cap}_{Q}\mathrm{h}\, \underline{\llbracket u = r\rrbracket}_{SR} \neq 0\rrbracket_{Q})
\end{array}
\tag{2}
$$

Схема 2.5.3

$$\frac{(2)}{\begin{aligned}
&V^{D}(G), V^{C}(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}v) \cap [\![G \cap \mathrm{h}\, [\![\underline{u \in v}]\!]_{S\,R} \neq 0]\!]_{Q} = \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}}\, {}_{\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap_{\overline{Q}}\, {}_{\overline{Q}} [\![\mathrm{h}\, \underline{v(r)}_{R} \in \mathrm{h}\, \underline{\mathrm{O}}_{R}(S)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R}\, \underline{\cap}_{Q} \mathrm{h}\, [\![\underline{u = r}]\!]_{S\,R} \neq 0]\!]_{Q}) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}}\, {}_{\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap_{\overline{Q}}\, {}_{\overline{Q}} [\![\mathrm{h}\, \underline{v(r)}_{R} \in \mathrm{h}\, \underline{\mathrm{O}}_{R}(S)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\, [\![\underline{u = r}]\!]_{S\,R} \neq 0]\!]_{Q}) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}}\, {}_{\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\, [\![\underline{u = r}]\!]_{S\,R} \neq 0]\!]_{Q}) \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}}\, {}_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\, [\![\underline{u = r}]\!]_{S\,R} \neq 0]\!]_{Q})
\end{aligned}}{\begin{aligned}
&V^{D}(G), V^{C}(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}v) \cap [\![G \cap \mathrm{h}\, [\![\underline{u \in v}]\!]_{S\,R} \neq 0]\!]_{Q} = \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}}\, {}_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\, \underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\, [\![\underline{u = r}]\!]_{S\,R} \neq 0]\!]_{Q})
\end{aligned}} \quad (3)$$

Схема 2.5.4

$$V^{D}(G), V^{C}(u,v,S) \vdash$$

$$\vdash a \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket u = v \rrbracket_{S R}} \neq 0 \rrbracket_{Q} =$$

$$= a \cap \llbracket G \cap \mathrm{h}\, \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{R} \underline{\cap}_{R} \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{R} \neq 0 \rrbracket_{Q} =$$

$$= a \cap \llbracket G \cap \mathrm{h}\, \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{R} \underline{\cap}_{Q} \mathrm{h}\, \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{R} \neq 0 \rrbracket_{Q} =$$

$$= a \cap \llbracket G \cap \mathrm{h}\, \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{R} \neq 0 \rrbracket_{Q} \cap$$

$$\cap \llbracket G \cap \mathrm{h}\, \underline{\overline{S}}_{R} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{R} \neq 0 \rrbracket_{Q} =$$

$$= a \cap \llbracket G \cap \underline{\overline{\mathrm{h}S}}_{Q} \mathrm{h} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{R} \neq 0 \rrbracket_{Q} \cap$$

$$\cap \llbracket G \cap \underline{\overline{\mathrm{h}S}}_{Q} \mathrm{h} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{R} \neq 0 \rrbracket_{Q} =$$

$$= a \cap \llbracket G \cap \mathrm{h} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{R} = 0 \rrbracket_{Q} \cap$$

$$\cap \llbracket G \cap \mathrm{h} \underline{\bigcup \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{R} = 0 \rrbracket_{Q} =$$

$$= a \cap \left\llbracket G \cap \underline{\bigcup \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(u,v,S)}_{Q} = 0 \right\rrbracket_{Q} \cap$$

$$\cap \left\llbracket G \cap \underline{\bigcup \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(v,u,S)}_{Q} = 0 \right\rrbracket_{Q} =$$

$$= a \cap \llbracket \neg \exists t \in \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(u,v,S)(G \cap t \neq 0) \rrbracket_{Q} \cap$$

$$\cap \llbracket \neg \exists t \in \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(v,u,S)(G \cap t \neq 0) \rrbracket_{Q} =$$

$$= a \cap \overline{Q} \bigcup_{V^{D}(t)} (\overline{Q}\, \overline{Q} \llbracket t \in \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(u,v,S) \rrbracket_{Q} \cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) \cap$$

$$\cap \overline{Q} \bigcup_{V^{D}(t)} (\overline{Q}\, \overline{Q} \llbracket t \in \mathrm{h}\, \underline{\mathrm{Ev}}_{\notin R}(v,u,S) \rrbracket_{Q} \cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) =$$

$$= a \cap \overline{Q} \bigcup_{V^{D}(t)} (\overline{Q}\, \overline{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(u) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \mathrm{h}\, \underline{u(r)}_{R} \underline{\cap}_{R} \underline{\overline{S}}_{R} \underline{\llbracket r \in v \rrbracket_{S R}} \rrbracket_{Q}) \cap$$

$$\cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) \cap$$

$$\cap \overline{Q} \bigcup_{V^{D}(t)} (\overline{Q}\, \overline{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{\overline{S}}_{R} \underline{\llbracket r \in u \rrbracket_{S R}} \rrbracket_{Q}) \cap$$

$$\cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) =$$

$$= a \cap \overline{Q} \bigcup_{V^{D}(t)} (\bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(u) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \mathrm{h}\, \underline{u(r)}_{R} \underline{\cap}_{R} \underline{\overline{S}}_{R} \underline{\llbracket r \in v \rrbracket_{S R}} \rrbracket_{Q}) \cap$$

$$\cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) \cap$$

$$\cap \overline{Q} \bigcup_{V^{D}(t)} (\bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(v) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \mathrm{h}\, \underline{v(r)}_{R} \underline{\cap}_{R} \underline{\overline{S}}_{R} \underline{\llbracket r \in u \rrbracket_{S R}} \rrbracket_{Q}) \cap$$

$$\cap \llbracket G \cap t \neq 0 \rrbracket_{Q}) =$$

$$
\begin{aligned}
= a \cap {}_{\overline{Q}} \bigcup_{V^D(t)} \bigcup_{V^C(r)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(u)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap t \neq 0 \rrbracket_Q) \cap \\
\cap {}_{\overline{Q}} \bigcup_{V^D(t)} \bigcup_{V^C(r)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(v)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap t \neq 0 \rrbracket_Q) = \\
= a \cap {}_{\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^D(t)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(u)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap t \neq 0 \rrbracket_Q) \cap \\
\cap {}_{\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^D(t)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(v)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap t \neq 0 \rrbracket_Q) = \\
= a \cap {}_{\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^D(t)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(u)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \neq 0 \rrbracket_Q) \cap \\
\cap {}_{\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^D(t)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(v)} \rrbracket_Q \cap \\
& \cap \llbracket t = \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \neq 0 \rrbracket_Q) = \\
= a \cap {}_{\overline{Q}} \bigcup_{V^C(r)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(u)} \rrbracket_Q \cap \\
& \cap \bigcup_{V^D(t)} \llbracket t = \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap \mathrm{h}\, \underline{u(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in v \rrbracket_{SR}} \neq 0 \rrbracket_Q) \cap \\
\cap {}_{\overline{Q}} \bigcup_{V^C(r)} & (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R(v)} \rrbracket_Q \cap \\
& \cap \bigcup_{V^D(t)} \llbracket t = \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \rrbracket_Q \cap \\
& \cap \llbracket G \cap \mathrm{h}\, \underline{v(r)_R} \,\underline{\cap}_R \,{}_{\overline{S}}{}_R \underline{\llbracket r \in u \rrbracket_{SR}} \neq 0 \rrbracket_Q) =
\end{aligned}
$$

$$= a \cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \ \underline{\cap}_R \ \underset{S}{-}_R \underline{\llbracket r \in v \rrbracket}_{SR} \neq 0 \rrbracket_Q) \cap$$

$$\cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \ \underline{\cap}_R \ \underset{S}{-}_R \underline{\llbracket r \in u \rrbracket}_{SR} \neq 0 \rrbracket_Q) =$$

$$= a \cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in v \rrbracket}_{SR} \neq 0 \rrbracket_Q) \cap$$

$$\cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in u \rrbracket}_{SR} \neq 0 \rrbracket_Q)$$

$$V^D(G), V^C(u, v, S) \vdash$$

$$\vdash a \cap \llbracket G \cap \mathrm{h}\ \underline{\llbracket u = v \rrbracket}_{SR} \neq 0 \rrbracket_Q =$$

$$= a \cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in v \rrbracket}_{SR} \neq 0 \rrbracket_Q) \cap$$

$$\cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in u \rrbracket}_{SR} \neq 0 \rrbracket_Q) \qquad (4)$$

(4)

$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \llbracket G \cap \mathrm{h}\ \underline{\llbracket u = v \rrbracket}_{SR} \neq 0 \rrbracket_Q =$$

$$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in v \rrbracket}_{SR} \neq 0 \rrbracket_Q) \cap$$

$$\cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in u \rrbracket}_{SR} \neq 0 \rrbracket_Q) =$$

$$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap$$

$$\cap \underset{Q}{-} \underset{Q}{-} \llbracket \mathrm{h}\ \underline{u(r)}_R \in \mathrm{h}\ \underline{\mathrm{O}}_R(S) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in v \rrbracket}_{SR} \neq 0 \rrbracket_Q) \cap$$

$$\cap \underset{Q}{-} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap$$

$$\cap \underset{Q}{-} \underset{Q}{-} \llbracket \mathrm{h}\ \underline{u(r)}_R \in \mathrm{h}\ \underline{\mathrm{O}}_R(S) \rrbracket_Q \cap$$

$$\cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \ \underline{\cap}_Q \mathrm{h}\ \underset{S}{-}_R \underline{\llbracket r \in u \rrbracket}_{SR} \neq 0 \rrbracket_Q) =$$

$$
\begin{aligned}
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap_{\overline{Q}}\ {}_{\overline{Q}} \llbracket \mathrm{h}\ \underline{u(r)}_R \in \mathrm{h}\ \underline{\mathrm{O}}_R(S) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ {}_{\underline{\overline{S}}\,R} \underline{\llbracket r \in v \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap_{\overline{Q}}\ {}_{\overline{Q}} \llbracket \mathrm{h}\ \underline{v(r)}_R \in \mathrm{h}\ \underline{\mathrm{O}}_R(S) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ {}_{\underline{\overline{S}}\,R} \underline{\llbracket r \in u \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ {}_{\underline{\overline{S}}\,R} \underline{\llbracket r \in v \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ {}_{\underline{\overline{S}}\,R} \underline{\llbracket r \in u \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap_{\underline{\overline{\mathrm{h}S}}\,Q} \mathrm{h}\ \underline{\llbracket r \in v \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap_{\underline{\overline{\mathrm{h}S}}\,Q} \mathrm{h}\ \underline{\llbracket r \in u \rrbracket_{S}}_{R} \neq 0 \rrbracket_Q) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{\llbracket r \in v \rrbracket_{S}}_{R} = 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\ \underline{\llbracket r \in u \rrbracket_{S}}_{R} = 0 \rrbracket_Q) =
\end{aligned}
$$

Схема 2.5.8

$$\begin{aligned}
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in v \rrbracket}_{SR} = 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in u \rrbracket}_{SR} = 0 \rrbracket_Q)
\end{aligned}$$

$$\begin{aligned}
&V^D(G), V^C(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket u = v \rrbracket}_{SR} \neq 0 \rrbracket_Q = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in v \rrbracket}_{SR} = 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in u \rrbracket}_{SR} = 0 \rrbracket_Q) \qquad (5)
\end{aligned}$$

(5)

$$\begin{aligned}
&V^D(G), V^C(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket u = v \rrbracket}_{SR} \neq 0 \rrbracket_Q = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \\
&\qquad \cap \llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in v \rrbracket}_{SR} = 0 \rrbracket_Q) \cap \\
&\quad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \\
&\qquad \cap \llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v) \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{v(r)}_R \neq 0 \rrbracket_Q \cap \\
&\qquad \cap \llbracket G \cap \mathrm{h}\, \underline{\llbracket r \in u \rrbracket}_{SR} = 0 \rrbracket_Q) \qquad (6)
\end{aligned}$$

Схема 2.5.9

$$V^D(G), V^C(r,v,S) \vdash$$

$$\vdash a \cap V_{R,S,Q}(\mathrm{h}v) \cap [\![G \cap \mathrm{h}\, \underline{[\![r \in v]\!]_{S\,R}} \neq 0]\!]_Q =$$

$$= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}\,\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{v(t)}_R \neq 0]\!]_Q$$

$$\cap [\![G \cap \mathrm{h}\, \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q) \quad \overset{u}{\underset{r}{\Longleftarrow}} (3);$$

$$V^D(G), V^C(r,u,S) \vdash$$

$$\vdash a \cap V_{R,S,Q}(\mathrm{h}v) \cap [\![G \cap \mathrm{h}\, \underline{[\![r \in u]\!]_{S\,R}} \neq 0]\!]_Q =$$

$$= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}\,\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{u(t)}_R \neq 0]\!]_Q$$

$$\cap [\![G \cap \mathrm{h}\, \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q) \quad \overset{u\ v}{\underset{r\ u}{\Longleftarrow}} (3); \quad (6)$$

$$V^D(G), V^C(u,v,S) \vdash$$

$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap [\![G \cap \mathrm{h}\, \underline{[\![u = v]\!]_{S\,R}} \neq 0]\!]_Q =$$

$$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap$$

$$\cap [\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{u(r)}_R \neq 0]\!]_Q \cap$$

$$\cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{v(t)}_R \neq 0]\!]_Q$$

$$\cap [\![G \cap \mathrm{h}\, \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q)) \cap$$

$$\cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap$$

$$\cap [\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{v(r)}_R \neq 0]\!]_Q \cap$$

$$\cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$

$$\cap [\![G \cap \mathrm{h}\, \underline{u(t)}_R \neq 0]\!]_Q$$

$$\cap [\![G \cap \mathrm{h}\, \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q)) =$$

$$
\begin{aligned}
&= a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap V_{R,S,Q}(\mathrm{h}r)\cap \\
&\qquad \cap [\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{u(r)}_R\neq 0]\!]_Q\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{v(t)}_R\neq 0]\!]_Q \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{[\![r=t]\!]}_{SR}\neq 0]\!]_Q))\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap V_{R,S,Q}(\mathrm{h}r)\cap \\
&\qquad \cap [\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{v(r)}_R\neq 0]\!]_Q\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{u(t)}_R\neq 0]\!]_Q \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{[\![r=t]\!]}_{SR}\neq 0]\!]_Q)) = \\
&= a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap V_{R,S,Q}(\mathrm{h}r)\cap \\
&\qquad \cap [\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{u(r)}_R\neq 0]\!]_Q\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a\cap V_{R,S,Q}(\mathrm{h}r,\mathrm{h}v)\cap \\
&\qquad \cap [\![\mathrm{h}t\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{v(t)}_R\neq 0]\!]_Q \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{[\![r=t]\!]}_{SR}\neq 0]\!]_Q))\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a\cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v)\cap V_{R,S,Q}(\mathrm{h}r)\cap \\
&\qquad \cap [\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{v(r)}_R\neq 0]\!]_Q\cap \\
&\quad \cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a\cap V_{R,S,Q}(\mathrm{h}r,\mathrm{h}u)\cap \\
&\qquad \cap [\![\mathrm{h}t\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{u(t)}_R\neq 0]\!]_Q \\
&\qquad \cap [\![G\cap \mathrm{h}\ \underline{[\![r=t]\!]}_{SR}\neq 0]\!]_Q)) =
\end{aligned}
$$

$$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}t) \cap$$
$$\cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q$$
$$\cap [\![G \cap \mathrm{h}\ \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q)) \cap$$
$$\cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap$$
$$\cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}u) \cap V_{R,S,Q}(\mathrm{h}t) \cap$$
$$\cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q$$
$$\cap [\![G \cap \mathrm{h}\ \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q));$$

$$\mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash$$
$$\vdash \forall r, t\ (r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))\ \&\ t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v)) \to$$
$$\to a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}t) \cap [\![G \cap \mathrm{h}\ \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}t) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)\ \&$$
$$\&\ \forall r, t\ (r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))\ \&\ t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(t)) \to$$
$$\to a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}t) \cap [\![G \cap \mathrm{h}\ \underline{[\![r = t]\!]_{S\,R}} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}t) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)$$

$$V^D(G), V^C(u, v, S), \mathrm{O}(\langle u, v \rangle, \mathfrak{R}) \subset' Y \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u = v]\!]_{S\,R}} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$

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$$
\begin{aligned}
&\cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}t) \cap \\
&\qquad \cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)) \cap \\
&\cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}u) \cap V_{R,S,Q}(\mathrm{h}t) \cap \\
&\qquad \cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap \\
&\cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}v) \cap \\
&\qquad \cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)) \cap \\
&\cap_{\overline{Q}} \bigcup_{r\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\cap_{\overline{Q}} \bigcup_{t\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}r, \mathrm{h}u) \cap \\
&\qquad \cap [\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, G) = \underline{\mathrm{In}}_Q(\mathrm{h}t, G)]\!]_Q)) =
\end{aligned}
$$

$$
\begin{aligned}
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} (a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) =
\end{aligned}
$$

$$
\begin{aligned}
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap_{\overline{Q}} \bigcup_{t \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}t \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(t)}_R \neq 0]\!]_Q \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) = \underline{\mathrm{In}}_Q(\mathrm{h}t,G)]\!]_Q)) = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) \notin \underline{\mathrm{In}}_Q(\mathrm{h}v,G)]\!]_Q) \\
&\qquad \cap_{\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r,G) \notin \underline{\mathrm{In}}_Q(\mathrm{h}u,G)]\!]_Q) \\
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u,G) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}v,G)]\!]_Q \cap \\
&\qquad \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}v,G) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}u,G)]\!]_Q = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u,G) = \underline{\mathrm{In}}_Q(\mathrm{h}v,G)]\!]_Q
\end{aligned}
$$

$$
\begin{aligned}
&V^D(G), V^C(u,v,S), \mathrm{O}(\langle u, v\rangle, \Re) \subset' Y \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u = v]\!]}_{SR} \neq 0]\!]_Q = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u,G) = \underline{\mathrm{In}}_Q(\mathrm{h}v,G)]\!]_Q \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
&V^{D}(G), V^{C}(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}v) \cap [\![G \cap \mathrm{h}\,\underline{[\![u \in v]\!]_{S}}_{R} \neq 0]\!]_{Q} = \\
&= a \cap V_{R,S,Q}(\mathrm{h}v) \cap_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} ([\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{[\![u = r]\!]_{S}}_{R} \neq 0]\!]_{Q}) \quad \Longleftarrow (3)
\end{aligned}
$$

$$
\begin{aligned}
&V^{D}(G), V^{C}(u,v,S) \vdash \\
&\vdash a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap [\![G \cap \mathrm{h}\,\underline{[\![u \in v]\!]_{S}}_{R} \neq 0]\!]_{Q} = \\
&= a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} ([\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{[\![u = r]\!]_{S}}_{R} \neq 0]\!]_{Q}) = \\
&= _{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{[\![u = r]\!]_{S}}_{R} \neq 0]\!]_{Q}) = \\
&= _{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{[\![u = r]\!]_{S}}_{R} \neq 0]\!]_{Q}) = {}^{1)} \\
&= _{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap V_{R,S,Q}(\mathrm{h}r) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}u, G) = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, G)]\!]_{Q}) = \\
&= _{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(v))} (a \cap V_{R,S,Q}(\mathrm{h}u, \mathrm{h}v) \cap \\
&\qquad \cap [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_{R}(v)]\!]_{Q} \cap \\
&\qquad \cap [\![G \cap \mathrm{h}\,\underline{v(r)}_{R} \neq 0]\!]_{Q} \\
&\qquad \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}u, G) = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, G)]\!]_{Q}) =
\end{aligned}
$$

$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}\,\underset{Q}{-} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(v))} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(v)]\!]_Q \cap$$
$$\cap [\![G \cap \mathrm{h}\ \underline{v(r)}_R \neq 0]\!]_Q$$
$$\cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) = \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q) =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}\,\underset{Q}{-} [\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \in \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q$$

$$V^D(G), V^C(u,v,S) \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}\,\underset{Q}{-} [\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \in \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q \qquad (8)$$

$$V^D(G), V^C(u,v,S) \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \notin v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underset{S}{-}\underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \underset{\mathrm{h}S}{-}\mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} = 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\neg\, G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}[\![G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}(a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q)$$

$$V^D(G), V^C(u,v,S) \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \notin v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}(a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \in v]\!]}_{SR} \neq 0]\!]_Q); \quad (8)$$

$$V^D(G), V^C(u,v,S) \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \notin v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap$$
$$\cap \underset{Q}{-}(a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}\,\underset{Q}{-}[\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \in \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q) =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap \underset{Q}{-}[\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \in \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \notin \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q$$

$$V^D(G), V^C(u,v,S) \vdash$$
$$\vdash a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![G \cap \mathrm{h}\ \underline{[\![u \notin v]\!]}_{SR} \neq 0]\!]_Q =$$
$$= a \cap V_{R,S,Q}(\mathrm{h}u,\mathrm{h}v) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}u, G) \notin \underline{\mathrm{In}}_Q(\mathrm{h}r, G)]\!]_Q \qquad (9)$$

Схема 2.5.17

$$\frac{V^D(r), V^C(S) \vdash |R| \subset' \overline{\ }_R \overline{\ }_R [\![\hat{r} \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[r])}}_R]\!]_R}{V^D(r), V^C(S) \vdash |Q| \subset' \overline{\ }_Q \overline{\ }_Q [\![\mathrm{h}\hat{r} \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[r])}}_R]\!]_Q \,;}$$

$$\frac{\begin{array}{l} V^D(r), V^C(S) \vdash \overline{\ }_Q \overline{\ }_Q [\![\mathrm{h}\hat{r} \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\mathrm{Sc}}(\mathrm{rg}[u])}}_R]\!]_Q \subset' \\ \qquad \subset' \overline{\ }_Q \overline{\ }_Q \bigcup\limits_{\mathrm{OR}(\alpha)} [\![\mathrm{h}\hat{r} \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_Q = V_{R,S,Q}(\mathrm{h}\hat{r}) \end{array}}{V^D(r), V^C(S) \vdash |Q| \subset' V_{R,S,Q}(\mathrm{h}\hat{r})\,;}$$

$$\frac{V^C(u,\hat{r}) \vdash V_{R,S,Q}(\mathrm{h}\hat{r}) \subset' [\![\mathrm{h}\, \underline{[\![\varphi(u,\hat{r})]\!]_S}_R \subset' \mathrm{h}\, \underline{[\![\exists v\, \varphi(u,v)]\!]_S}_R]\!]_Q}{V^D(r), V^C(u,S) \vdash |Q| \subset' [\![\mathrm{h}\, \underline{[\![\varphi(u,\hat{r})]\!]_S}_R \subset' \mathrm{h}\, \underline{[\![\exists v\, \varphi(u,v)]\!]_S}_R]\!]_Q} \qquad (1)$$

$$\frac{V^D(r), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q}{\begin{array}{l} V^D(r), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash |Q| \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset' \\ \qquad \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \end{array}}$$

$$\begin{array}{l} V^D(r), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset' \\ \qquad \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \,; \end{array}$$

$$\frac{\begin{array}{l} V^D(r,u), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash \\ \qquad \vdash [\![\underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)) = r]\!]_Q \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset' \\ \qquad \subset' [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \end{array}}{\begin{array}{l} V^D(r,u), V^C(S), G = \mathrm{gn}(R \otimes'_{\Delta} S) \vdash [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset' \\ \qquad \subset' [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \end{array}} \qquad (2)$$

$$\begin{array}{l} V^C(u,r,S) \vdash \overline{\ }_R \overline{\ }_R \Big[\!\Big[r \in \underline{\Big\| \varphi(u,.) \Big\|^S}_R \Big]\!\Big]_R = \\ = \overline{\ }_R \overline{\ }_R [\![\exists \alpha (\mathrm{OR}(\alpha)\, \exists v (\neg\neg\, v \in \mathrm{V}^{\mathrm{O}(S)}_{\alpha} \;\&\; r = [\![\varphi(u,v)]\!]_S))]\!]_R = \\ = \overline{\ }_R \overline{\ }_R \bigcup\limits_{\mathrm{OR}(\alpha)} [\![\exists v (\neg\neg\, v \in \mathrm{V}^{\mathrm{O}(S)}_{\check{\alpha}} \;\&\; r = [\![\varphi(u,v)]\!]_S)]\!]_R = \\ = \overline{\ }_R \overline{\ }_R \bigcup\limits_{\mathrm{OR}(\alpha)} \bigcup\limits_{V^C(v)} [\![\neg\neg\, v \in \mathrm{V}^{\mathrm{O}(S)}_{\check{\alpha}} \;\&\; r = [\![\varphi(u,v)]\!]_S]\!]_R = \\ = \overline{\ }_R \overline{\ }_R \bigcup\limits_{\mathrm{OR}(\alpha)} \bigcup\limits_{V^C(v)} ([\![\neg\neg\, v \in \mathrm{V}^{\mathrm{O}(S)}_{\check{\alpha}}]\!]_R \cap [\![r = [\![\varphi(u,v)]\!]_S]\!]_R) = \\ = \overline{\ }_R \overline{\ }_R \bigcup\limits_{\mathrm{OR}(\alpha)} \bigcup\limits_{V^C(v)} (\overline{\ }_R \overline{\ }_R [\![v \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_R \cap [\![r = [\![\varphi(u,v)]\!]_S]\!]_R) = \\ = \overline{\ }_R \overline{\ }_R \bigcup\limits_{\mathrm{OR}(\alpha)} \bigcup\limits_{V^C(v)} ([\![v \in \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_R \cap [\![r = [\![\varphi(u,v)]\!]_S]\!]_R) = \end{array}$$

$$= \overline{\ }_R\,\overline{\ }_R \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket v \in \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket r = \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R) =$$

$$\frac{}{V^C(u,r,S) \vdash \overline{\ }_R\,\overline{\ }_R \left\llbracket r \in \underline{\left\| \varphi(u,.) \right\|^S}_R \right\rrbracket_R =}$$

$$= \overline{\ }_R\,\overline{\ }_R \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket v \in \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket r = \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R) \qquad (3)$$

$$\frac{(3)}{V^C(u,r,S) \vdash \overline{\ }_Q\,\overline{\ }_Q \left\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\left\| \varphi(u,.) \right\|^S}_R \right\rrbracket_Q =}$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_Q \cap \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_Q) \qquad (4)$$

$$V^D(G), V^C(u,S) \vdash$$

$$\vdash \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}\, \underline{\llbracket \exists v\, \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_Q =$$

$$= \left\llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h} \underline{\bigcup \underline{\left\| \varphi(u,.) \right\|^S}_R}_R \neq 0 \right\rrbracket_Q =$$

$$= \left\llbracket \underline{\mathrm{Rg}_Q}(G) \cap \underline{\bigcup \mathrm{h}\, \underline{\left\| \varphi(u,.) \right\|^S}_R}_Q \neq 0 \right\rrbracket_Q =$$

$$= \left\llbracket \neg\neg \exists t \in \mathrm{h}\, \underline{\left\| \varphi(u,.) \right\|^S}_R (\underline{\mathrm{Rg}_Q}(G) \cap t \neq 0) \right\rrbracket_Q =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \left(\left\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\left\| \varphi(u,.) \right\|^S}_R \right\rrbracket_Q \cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}r \neq 0 \rrbracket_Q \right) =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \left(\overline{\ }_Q\,\overline{\ }_Q \left\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\left\| \varphi(u,.) \right\|^S}_R \right\rrbracket_Q \cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}r \neq 0 \rrbracket_Q \right) =$$ [1)]

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \left(\overline{\ }_R\,\overline{\ }_R \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R) \cap \right.$$

$$\left. \cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}r \neq 0 \rrbracket_Q \right) =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \left(\bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R) \cap \right.$$

$$\left. \cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}r \neq 0 \rrbracket_Q \right) =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}r \neq 0 \rrbracket_Q) =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(r)} \bigcup_{\mathrm{OR}(\alpha)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_Q) =$$

$$= \overline{\ }_Q\,\overline{\ }_Q \bigcup_{V^C(v)} \bigcup_{\mathrm{OR}(\alpha)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{Q}_R(S)}_{\breve{\alpha}}}_R \rrbracket_R \cap \bigcup_{V^C(r)} \llbracket \mathrm{h}r = \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \rrbracket_R \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q) =$$

$$= \overline{Q}\,\overline{Q} \bigcup_{V^C(v)} \bigcup_{\mathrm{OR}(\alpha)} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_R \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q) =$$

$$= \overline{Q}\,\overline{Q} \bigcup_{V^C(v)} (\bigcup_{\mathrm{OR}(\alpha)} [\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_R \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q) =$$

$$= \overline{Q}\,\overline{Q} \bigcup_{V^C(v)} (\overline{Q}\,\overline{Q} \bigcup_{\mathrm{OR}(\alpha)} [\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{V}^{\mathrm{O}_R(S)}_{\check{\alpha}}}_R]\!]_R \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q) =$$

$$= \overline{Q}\,\overline{Q} \bigcup_{V^C(v)} (V_{R,S,Q}(\mathrm{h}v) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q)$$

$$V^D(G), V^C(u,S) \vdash$$

$$\vdash [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\exists v\, \varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q =$$

$$= \overline{Q}\,\overline{Q} \bigcup_{V^C(v)} (V_{R,S,Q}(\mathrm{h}v) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,v)]\!]}_{SR} \neq 0]\!]_Q) \qquad (5)$$

$$V^D(r), V^C(S) \vdash |Q| \subset' V_{R,S,Q}(\mathrm{h}\hat{r}); \qquad (2)$$

$$V^D(r), V^C(u,S), G = \mathrm{gn}(R \otimes'_\Delta S) \vdash |Q| \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset'$$

$$\subset' V_{R,S,Q}(\mathrm{h}\hat{r}) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q$$

$$V^D(r), V^C(u,S), G = \mathrm{gn}(R \otimes'_\Delta S) \vdash [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset'$$

$$\subset' V_{R,S,Q}(\mathrm{h}\hat{r}) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q$$

$$V^D(r), V^C(u,S), G = \mathrm{gn}(R \otimes'_\Delta S) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset'$$

$$\subset' V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\hat{r}) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q;$$

$$V^C(u,\hat{r}), \forall y,t\, (V^C(y,t) \to$$

$$\to V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}y, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}t, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q =$$

$$= V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(y,t)]\!]}_{SR} \neq 0]\!]_Q) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\hat{r}) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}\hat{r}, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q =$$

$$= V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\hat{r}) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,\hat{r})]\!]}_{SR} \neq 0]\!]_Q$$

$$V^D(r), V^C(u,S), V^C(\hat{r}), G = \mathrm{gn}(R \otimes'_\Delta S), \forall y,t\, (V^C(y,t) \to$$

$$\to V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}y, \underline{\mathrm{Rg}}_Q(G)), \underline{\mathrm{In}}_Q(\mathrm{h}t, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q =$$

$$= V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(y,t)]\!]}_{SR} \neq 0]\!]_Q) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![\varphi(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)), r)]\!]_Q \subset'$$

$$\subset' V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\hat{r}) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{[\![\varphi(u,\hat{r})]\!]}_{SR} \neq 0]\!]_Q \qquad (6)$$

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$$(6)\,;$$

$$V^{D}(r), V^{C}(u,S) \vdash |Q| \subset' \llbracket \mathrm{h}\, \llbracket \underline{\varphi(u,\hat{r})} \rrbracket_{SR} \subset' \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \rrbracket_{Q}$$

$$\begin{aligned}
&V^{D}(r), V^{C}(u,S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y,t\, (V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y, \underline{\mathrm{Rg}}_{Q}(G)), \underline{\mathrm{In}}_{Q}(\mathrm{h}t, \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\varphi(y,t)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\vdash |Q| \cap V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), r) \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\varphi(u,\hat{r})} \rrbracket_{SR} \neq 0 \rrbracket_{Q} \cap \\
&\qquad\qquad \cap \llbracket \mathrm{h}\, \llbracket \underline{\varphi(u,\hat{r})} \rrbracket_{SR} \subset' \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}
\end{aligned}$$

$$\begin{aligned}
&V^{D}(r), V^{C}(u,S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y,t\, (V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y, \underline{\mathrm{Rg}}_{Q}(G)), \underline{\mathrm{In}}_{Q}(\mathrm{h}t, \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\varphi(y,t)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), r) \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}
\end{aligned}$$

$$\begin{aligned}
&\ V^{C}(u,S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y,t\, (V^{C}(y,t) \to \dots) \vdash \forall r\, (V^{D}(r) \to \\
&\to V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), r) \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \neq 0 \rrbracket_{Q})
\end{aligned}$$

$$\begin{aligned}
&\ V^{C}(u,S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y,t\, (V^{C}(y,t) \to \dots) \vdash \\
&\vdash V_{R,S,Q}(\mathrm{h}u) \cap \bigcup_{V^{D}(r)} \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), r) \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}
\end{aligned}$$

$$\begin{aligned}
&\ V^{C}(u,S), G = \mathrm{gn}(R \otimes'_{\Delta} S), \forall y,t\, (V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y, \underline{\mathrm{Rg}}_{Q}(G)), \underline{\mathrm{In}}_{Q}(\mathrm{h}t, \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y, \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\varphi(y,t)} \rrbracket_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)), r) \rrbracket_{Q} \subset' \\
&\quad \subset' \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \llbracket \underline{\exists v\, \varphi(u,v)} \rrbracket_{SR} \neq 0 \rrbracket_{Q} \qquad (7)
\end{aligned}$$

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$$
\begin{aligned}
& V^{C}(u,v),\ \forall y,t\,(V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y,\ \underline{\mathrm{Rg}}_{Q}(G)),\ \underline{\mathrm{In}}_{Q}(\mathrm{h}t,\ \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(y,t) \rrbracket}_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\qquad \vdash V_{R,S,Q}(\mathrm{h}u,\ \mathrm{h}v) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q} \subset' \\
&\qquad \subset' \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ \underline{\mathrm{In}}_{Q}(\mathrm{h}v,\ \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q}\,;
\end{aligned}
$$

$$
\begin{aligned}
& V^{D}(\underline{\mathrm{In}}_{Q}(\mathrm{h}v,\ \underline{\mathrm{Rg}}_{Q}(G))) \vdash \\
&\qquad \vdash \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ \underline{\mathrm{In}}_{Q}(\mathrm{h}v,\ \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} \subset' \\
&\qquad \subset' \bigcup_{V^{D}(r)} \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q} = \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q}
\end{aligned}
$$

$$
\begin{aligned}
& V^{D}(G),\ V^{C}(u,v),\ \forall y,t\,(V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y,\ \underline{\mathrm{Rg}}_{Q}(G)),\ \underline{\mathrm{In}}_{Q}(\mathrm{h}t,\ \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(y,t) \rrbracket}_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\qquad \vdash V_{R,S,Q}(\mathrm{h}u,\ \mathrm{h}v) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q} \subset' \\
&\qquad \subset' \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q}
\end{aligned}
$$

$$
\begin{aligned}
& V^{D}(G),\ V^{C}(u),\ \forall y,t\,(V^{C}(y,t) \to \ldots\,) \vdash \\
& \vdash \forall v\,(\,V^{C}(v) \to V_{R,S,Q}(\mathrm{h}u) \cap V_{R,S,Q}(\mathrm{h}v) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q} \subset' \\
&\qquad \subset' \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q}\,)
\end{aligned}
$$

$$
\begin{aligned}
& V^{D}(G),\ V^{C}(u),\ \forall y,t\,(V^{C}(y,t) \to \ldots\,) \vdash \\
&\qquad \vdash V_{R,S,Q}(\mathrm{h}u) \cap \bigcup_{V^{C}(v)} (V_{R,S,Q}(\mathrm{h}v) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q}) \subset' \\
&\qquad \subset' \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q}
\end{aligned}
$$

$$
\begin{aligned}
& V^{D}(G),\ V^{C}(u),\ \forall y,t\,(V^{C}(y,t) \to \ldots\,) \vdash \\
&\quad \vdash V_{R,S,Q}(\mathrm{h}u) \cap_{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(v)} (V_{R,S,Q}(\mathrm{h}v) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q}) \subset' \\
&\qquad \subset'_{\overline{Q}\,\overline{Q}} \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q}
\end{aligned}
$$

(5);

$$
\begin{aligned}
& V^{D}(G),\ V^{C}(u),\ \forall y,t\,(V^{C}(y,t) \to \\
&\qquad \to V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}y,\ \underline{\mathrm{Rg}}_{Q}(G)),\ \underline{\mathrm{In}}_{Q}(\mathrm{h}t,\ \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} = \\
&\qquad = V_{R,S,Q}(\mathrm{h}y,\ \mathrm{h}t) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \varphi(y,t) \rrbracket}_{SR} \neq 0 \rrbracket_{Q}) \vdash \\
&\quad \vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\llbracket \exists v\, \varphi(u,v) \rrbracket}_{SR} \neq 0 \rrbracket_{Q} \\
&\qquad \subset'_{\overline{Q}\,\overline{Q}} \llbracket \exists r\, \varphi(\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\ \underline{\mathrm{Rg}}_{Q}(G)),\ r) \rrbracket_{Q} \qquad (8)
\end{aligned}
$$

Схема 2.6.1

$$V^{C}(t), V^{C}(u), V^{C}(S) \vdash_{\overline{R}\,\overline{R}} [\![t \in \underline{\mathrm{P}}_{SR}(u)]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} (\underline{\mathrm{P}}_{SR}(u)(v) \cap [\![t = v]\!]_R) =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} ([\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(v = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap [\![t = v]\!]_R) =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} ([\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap [\![t = v]\!]_R) =$$

$$=_{\overline{R}\,\overline{R}} ([\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R) =$$

$$=_{\overline{R}\,\overline{R}} [\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

$$= [\![\neg\neg\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

$$= [\![\neg\neg\exists y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap$$
$$\cap [\![t = \langle y, [\![y \subset' u]\!]_S \rangle]\!]_R) \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap$$
$$\cap [\![y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \cap$$
$$\cap [\![t = \langle y, [\![y \subset' u]\!]_S \rangle]\!]_R) \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

$$=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap$$
$$\cap [\![y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \cap$$
$$\cap [\![t = \langle y, \underline{[\![y \subset' u]\!]_{SR}} \rangle]\!]_R) \cap$$
$$\cap_{\overline{R}\,\overline{R}} \bigcup_{v \in \mathrm{Dom}(\underline{\mathrm{P}}_{SR}(u))} [\![t = v]\!]_R =$$

Схема 2.6.2

$$= \overline{\;}_R\, \overline{\;}_R \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))(y) \cap$$

$$\cap [\![y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \underline{[\![y \subset' u]\!]}_{SR} \in \underline{\mathrm{P}}_R(\underline{|S|}_R) \cap$$

$$\cap [\![t = \langle y, \underline{[\![y\subset' u]\!]}_{SR}\rangle]\!]_R) \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{v\in\mathrm{Dom}(\underline{\mathrm{P}}_{\underline{S}R}(u))} [\![t=v]\!]_R =$$

$$= \overline{\;}_R\, \overline{\;}_R \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))(y) \cap$$

$$\cap [\![y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \underline{[\![y \subset' u]\!]}_{SR} \in \underline{\mathrm{P}}_R(\underline{|S|}_R) \cap$$

$$\cap [\![t = \langle y, \underline{[\![y\subset' u]\!]}_{SR}\rangle]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R [\![t \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))\underline{\times}_R\underline{\mathrm{P}}_R(\underline{|S|}_R)]\!]_R) \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{v\in\mathrm{Dom}(\underline{\mathrm{P}}_{\underline{S}R}(u))} [\![t=v]\!]_R =$$

$$= \overline{\;}_R\, \overline{\;}_R \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))(y) \cap$$

$$\cap [\![y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))]\!]_R \cap$$

$$\cap [\![t = \langle y, \underline{[\![y\subset' u]\!]}_{SR}\rangle]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R [\![t \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))\underline{\times}_R\underline{\mathrm{P}}_R(\underline{|S|}_R)]\!]_R) \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{v\in\mathrm{Dom}(\underline{\mathrm{P}}_{\underline{S}R}(u))} [\![t=v]\!]_R =$$

$$= \overline{\;}_R\, \overline{\;}_R \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))(y) \cap$$

$$\cap [\![t = \langle y, \underline{[\![y\subset' u]\!]}_{SR}\rangle]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R [\![t \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))\underline{\times}_R\underline{\mathrm{P}}_R(\underline{|S|}_R)]\!]_R) \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{v\in\mathrm{Dom}(\underline{\mathrm{P}}_{\underline{S}R}(u))} [\![t=v]\!]_R =$$

$$= \overline{\;}_R\, \overline{\;}_R [\![t \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))\underline{\times}_R\underline{\mathrm{P}}_R(\underline{|S|}_R)]\!]_R \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))(y) \cap$$

$$\cap [\![t = \langle y, \underline{[\![y\subset' u]\!]}_{SR}\rangle]\!]_R) \cap$$

$$\cap\, \overline{\;}_R\, \overline{\;}_R \bigcup_{v\in\mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\underline{\times}_R\underline{\mathrm{O}}_R(S))\underline{\times}_R\underline{\mathrm{P}}_R(\underline{|S|}_R))} [\![t=v]\!]_R =$$

Схема 2.6.3

$$
\begin{aligned}
&=_{\overline{R}\,\overline{R}} [\![t \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)) \underline{\times}_R \underline{\mathrm{P}}_R(\underline{|S|}_R)]\!]_R \cap \\
&\cap_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![t = \langle y, \underline{[\![y \subset' u]\!]_{SR}} \rangle]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![t = \langle y, \underline{[\![y \subset' u]\!]_{SR}} \rangle]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![t = \langle y, [\![y \subset' u]\!]_S \rangle]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} [\![\exists y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} [\![\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R
\end{aligned}
$$

$$
\begin{aligned}
&V^C(t), V^C(u)\, V^C(S) \vdash_{\overline{R}\,\overline{R}} [\![t \in \underline{\underline{\mathrm{P}}}_{SR}(u)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} [\![\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(t = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&V^C(t), V^C(v), V^C(u)\, V^C(S) \vdash_{\overline{R}\,\overline{R}} [\![\underline{\langle t, v \rangle}_R \in \underline{\underline{\mathrm{P}}}_{SR}(u)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} [\![\exists y \in \mathrm{PF}(\mathrm{Dom}(u) \times \mathrm{O}(S))(\underline{\langle t, v \rangle}_R = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} [\![\exists y \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(\underline{\langle t, v \rangle}_R = \langle y, [\![y \subset' u]\!]_S \rangle)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![\underline{\langle t, v \rangle}_R = \langle y, [\![y \subset' u]\!]_S \rangle]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![\underline{\langle t, v \rangle}_R = \langle y, \underline{[\![y \subset' u]\!]_{SR}} \rangle]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![\underline{\langle t, v \rangle}_R = \underline{\langle y, \underline{[\![y \subset' u]\!]_{SR}} \rangle}_R]\!]_R) = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![t = y]\!]_R \cap [\![v = \underline{[\![y \subset' u]\!]_{SR}}]\!]_R) =
\end{aligned}
$$

$$
\begin{aligned}
&V^C(t), V^C(v), V^C(u)\, V^C(S) \vdash_{\overline{R}\,\overline{R}} [\![\underline{\langle t, v \rangle}_R \in \underline{\underline{\mathrm{P}}}_{SR}(u)]\!]_R = \\
&=_{\overline{R}\,\overline{R}} \bigcup_{y \in \mathrm{Dom}(\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S)))} (\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u) \underline{\times}_R \underline{\mathrm{O}}_R(S))(y) \cap \\
&\qquad\qquad \cap [\![t = y]\!]_R \cap [\![v = \underline{[\![y \subset' u]\!]_{SR}}]\!]_R) \quad (2)
\end{aligned}
$$

Схема 2.6.4

$$
\begin{aligned}
& V^{C}(u,S), V^{D}(t) \vdash \\
& \vdash \underset{Q}{-}\,\underset{Q}{-}[\![t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} = \\
& = \underset{Q}{-}\,\underset{Q}{-} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(u)(r)}_{R} \neq 0]\!]_{Q} \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) = \\
& = \underset{Q}{-}\,\underset{Q}{-} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap \\
& \qquad \cap [\![\mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(u)(r)}_{R} = \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}}]\!]_{R} \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(u)(r)}_{R} \neq 0]\!]_{Q} \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) = \\
& = \underset{Q}{-}\,\underset{Q}{-} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap \\
& \qquad \cap [\![\mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(u)(r)}_{R} = \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}}]\!]_{R} \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}} \neq 0]\!]_{Q} \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) = \\
& = \underset{Q}{-}\,\underset{Q}{-} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}} \neq 0]\!]_{Q} \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q})
\end{aligned}
$$

$$
\begin{aligned}
& V^{C}(u,S), V^{D}(t) \vdash \\
& \vdash \underset{Q}{-}\,\underset{Q}{-}[\![t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} = \\
& = \underset{Q}{-}\,\underset{Q}{-} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap \\
& \qquad \cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}} \neq 0]\!]_{Q} \cap \\
& \qquad \cap [\![t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) \qquad (1)
\end{aligned}
$$

(1)

$$V^{C}(u,S), V^{D}(t) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap_{\overline{Q}\,\overline{Q}} [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap_{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{[\![r \subset' u]\!]}_{SR} \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$= _{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$

$$\cap V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{[\![r \subset' u]\!]}_{SR} \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$= _{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$

$$\cap V_{R,S,Q}(\mathrm{h}u) \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{[\![r \subset' u]\!]}_{SR} \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) = \ ^{1)}$$

$$\cap_{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$

$$\cap V_{R,S,Q}(\mathrm{h}u) \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$\cap_{\overline{Q}\,\overline{Q}} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$

$$\cap V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$= \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} (V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q})) =$$

$$= \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q}) =$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q}) =$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q}) =$$

$$= V_{R,S,Q}(\mathrm{h}u)_{Q} \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} (\llbracket t \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q})) =$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \llbracket t \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q} \cap$$

$$\cap \overline{\;\;}_{Q}\, \overline{\;\;}_{Q} \bigcup_{V^{C}(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(\underline{\mathrm{P}}_{S\,R}(u)) \rrbracket_{Q} \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \rrbracket_{Q}) =$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap \overline{\ }_Q \overline{\ }_Q [\![t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \cap$$
$$\cap \overline{\ }_Q \overline{\ }_Q \bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$$

$$V^C(u,S), V^D(t) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap \overline{\ }_Q \overline{\ }_Q [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q =$$
$$= V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap \overline{\ }_Q \overline{\ }_Q [\![t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \cap$$
$$\cap \overline{\ }_Q \overline{\ }_Q \bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \qquad (3)$$

$$(3)$$

$$V^C(u,S), V^D(t) \vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q \subset' \overline{\ }_Q \overline{\ }_Q [\![t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q$$

$$V^C(u,S), V^D(t) \vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap \overline{\ }_Q [\![t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset' 0$$

$$V^C(u,S), V^D(t) \vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G)) \ \&\ \neg\, t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset' 0$$

$$V^C(u,S) \vdash \forall t\, (V^D(t) \to V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G)) \ \&\ \neg\, t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset' 0)$$

$$V^C(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap \bigcup_{V^D(t)} [\![t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G)) \ \&\ \neg\, t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G)))]\!]_Q \subset' 0$$

$$V^C(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \cap$$
$$\cap [\![\exists t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_Q(G))\, (\neg\, t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))))]\!]_Q \subset' 0 \ (4)$$

Схема 2.6.8

$$
\begin{array}{l}
(4) \Longrightarrow \quad V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \cap \\
\cap \llbracket \exists t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G)) \, (\neg\, t \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))) \rrbracket_{Q} \subset' 0 \\
\hline
\quad V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' \\
\subset' \overline{}_{Q} \llbracket \exists t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G)) \, (\neg\, t \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))) \rrbracket_{Q} \\
= \llbracket \neg \exists t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G)) \, (\neg\, t \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))) \rrbracket_{Q} = \\
= \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G)) \subset' \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} \\
\hline
\quad V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' \\
\subset' \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G)) \subset' \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))) \rrbracket_{Q} \qquad (5)
\end{array}
$$

$$V^C(t), V^C(u), V^C(S) \vdash [\![t \in u_{\cap Z}]\!]_R =$$
$$= \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} (u_{\cap Z}(r) \cap [\![t = r]\!]_R) =$$
$$= \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} (\bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[r = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R) \cap$$
$$\cap [\![t = r]\!]_R) =$$
$$= \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[r = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R \cap$$
$$\cap [\![t = r]\!]_R) =$$
$$= \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R \cap$$
$$\cap [\![t = r]\!]_R) =$$
$$= \bigcup_{V^C(v)} \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R \cap$$
$$\cap [\![t = r]\!]_R) =$$
$$= \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R \cap$$
$$\cap \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} [\![t = r]\!]_R)$$

$$V^C(t), V^C(u), V^C(S) \vdash [\![t \in u_{\cap Z}]\!]_R =$$
$$= \bigcup_{V^C(v)} ([\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \left[\!\!\left[t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(} [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \right]\!\!\right]_R \cap$$
$$\cap \bigcup_{r \in \mathrm{Dom}(u_{\cap Z})} [\![t = r]\!]_R) \qquad (1)$$

Схема 2.6.10

$$
\begin{array}{l}
\vdash |R| \subset' [\![\, \underline{u(v)}_R \subset' \bigcup \underline{\mathrm{Rg}_R(u)_R}]\!]_R \subset' \\
\qquad \subset' [\![\, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \subset' \bigcup \underline{\mathrm{Rg}_R(u)_R}]\!]_R \subset' \\
\qquad \subset' [\![\, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \in \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R \\
\hline
\vdash |R| \subset' [\![\, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \in \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R ; \\
V^C(v), V^C(u), V^C(t), V^C(S) \vdash \\
\qquad \vdash [\![\, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \in \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R \cap \\
\qquad \cap [\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \\
\qquad \cap [\![t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle]\!]_R \subset' \\
\qquad \subset' [\![t \in \underline{\mathrm{Dom}}_R(u) \,\underline{\times}_R \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R \\
\hline
V^C(v), V^C(u), V^C(t), V^C(S) \vdash \\
\qquad \vdash [\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \\
\qquad \cap [\![t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle]\!]_R \subset' \\
\qquad \subset' [\![t \in \underline{\mathrm{Dom}}_R(u) \,\underline{\times}_R \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R ; \\
V^C(u), V^C(t) \vdash \\
\qquad \vdash [\![t \in \underline{\mathrm{Dom}}_R(u) \,\underline{\times}_R \underline{\mathrm{P}}_R(\bigcup \underline{\mathrm{Rg}_R(u)_R})]\!]_R \subset' \bigcup\limits_{r \in \mathrm{Dom}(u \cap Z)} [\![t = r]\!]_R \\
\hline
V^C(v), V^C(u), V^C(t), V^C(S) \vdash \\
\qquad \vdash [\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \\
\qquad \cap [\![t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle]\!]_R \subset' \\
\qquad \subset' \bigcup\limits_{r \in \mathrm{Dom}(u \cap Z)} [\![t = r]\!]_R \\
\hline
\qquad V^C(u), V^C(t), V^C(S) \vdash \forall v\, (V^C(v) \to \\
\qquad \to [\![v \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \\
\qquad \cap [\![t = \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle]\!]_R \subset' \\
\qquad \subset' \bigcup\limits_{r \in \mathrm{Dom}(u \cap Z)} [\![t = r]\!]_R) \qquad\qquad (2)
\end{array}
$$

Схема 2.6.11

$$(2)\Longrightarrow\quad V^C(u),V^C(t),V^C(S)\vdash\forall v\,(V^C(v)\to\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap$$
$$\cap\Big\llbracket t=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\subset'$$
$$\subset'\bigcup_{r\in\mathrm{Dom}(u_{\cap Z})}\llbracket t=r\rrbracket_R\,);$$
$$V^C(t),V^C(u),V^C(S)\vdash\llbracket t\in u_{\cap Z}\rrbracket_R=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\Big\llbracket t=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\cap$$
$$\cap\bigcup_{r\in\mathrm{Dom}(u_{\cap Z})}\llbracket t=r\rrbracket_R\,)\Longleftarrow(1)$$

$$V^C(t),V^C(u),V^C(S)\vdash\llbracket t\in u_{\cap Z}\rrbracket_R=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\Big\llbracket t=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\,)\qquad(3)$$

$$(3)\underset{\underline{\langle r,t\rangle}_R}{\overset{t}{\Longrightarrow}}\quad V^C(\underline{\langle r,t\rangle}_R),V^C(u),V^C(S)\vdash\llbracket\underline{\langle r,t\rangle}_R\in u_{\cap Z}\rrbracket_R=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\Big\llbracket\underline{\langle r,t\rangle}_R=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\,)$$

$$V^C(r,t),V^C(u),V^C(S)\vdash\llbracket\underline{\langle r,t\rangle}_R\in u_{\cap Z}\rrbracket_R=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\Big\llbracket\underline{\langle r,t\rangle}_R=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\,)$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\Big\llbracket\langle r,t\rangle=\langle v,\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\rangle\Big\rrbracket_R\,)=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\llbracket r=v\rrbracket_R\cap\Big\llbracket t=\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\Big\rrbracket_R\,)$$

$$V^C(r,t),V^C(u),V^C(S)\vdash\llbracket\underline{\langle r,t\rangle}_R\in u_{\cap Z}\rrbracket_R=$$
$$=\bigcup_{V^C(v)}(\llbracket v\in\underline{\mathrm{Dom}}_R(u)\rrbracket_R\cap\mathrm{Dom}(\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q)\cap$$
$$\cap\llbracket r=v\rrbracket_R\cap\Big\llbracket t=\underline{u(v)}_R\,\underline{\cap}_R\mathrm{Sh}\mathring{(}\,\llbracket\underline{\mathrm{In}}_Q(\mathrm{h}v,\underline{\mathrm{Rg}}_Q(G))\in Z\rrbracket_Q,R,S)\Big\rrbracket_R\,)\qquad(4)$$

Схема 2.6.12

$$V^{C}(r,v,u,S), V^{D}(Z) \vdash$$
$$\vdash [\![\mathrm{h}r = \mathrm{h}v]\!]_{Q} \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q} \subset' [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q} ;$$
$$[\![\mathrm{h}r = \mathrm{h}v]\!]_{Q} \cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q} \subset' [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q} ,$$
$$V^{C}(r,v,u,S), V^{D}(Z) \vdash [\![r = v]\!]_{R} \subset'$$
$$\subset' \Big\| \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S)$$
$$= \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big\|_{R} \quad ^{1)}$$

$$V^{C}(r,v,u,S), V^{D}(Z) \vdash [\![r = v]\!]_{R} \subset'$$
$$\subset' \Big\| \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S)$$
$$= \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big\|_{R} ;$$
$$V^{C}(r,v,u) \vdash [\![r = v]\!]_{R} \subset' [\![\underline{u(r)}_{R} = \underline{u(v)}_{R}]\!]_{R}$$

$$V^{C}(r,v,u,S), V^{D}(Z) \vdash [\![r = v]\!]_{R} \subset'$$
$$\subset' \Big\| \underline{u(r)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S)$$
$$= \underline{u(v)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big\|_{R}$$

$$V^{C}(r,u,S), V^{D}(Z) \vdash \forall v\, ([\![r = v]\!]_{R} \subset'$$
$$\subset' \Big\| \underline{u(r)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S)$$
$$= \underline{u(v)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big\|_{R}) \qquad (5)$$

$$(4) \Longrightarrow \quad V^{C}(r,t,u,S) \vdash [\![\underline{\langle r, t \rangle}_{R} \in u_{\cap Z}]\!]_{R} =$$
$$= \bigcup_{V^{C}(v)} ([\![v \in \underline{\mathrm{Dom}}_{R}(u)]\!]_{R} \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}) \cap$$
$$\cap [\![r = v]\!]_{R} \cap \Big[\!\!\Big[t = \underline{u(v)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big]\!\!\Big]_{R}) ; \qquad (5)$$

$$V^{C}(r,t,u,S), V^{D}(Z) \vdash [\![\underline{\langle r, t \rangle}_{R} \in u_{\cap Z}]\!]_{R} =$$
$$= \bigcup_{V^{C}(v)} ([\![v \in \underline{\mathrm{Dom}}_{R}(u)]\!]_{R} \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}) \cap$$
$$\cap [\![r = v]\!]_{R} \cap \Big[\!\!\Big[t = \underline{u(r)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big]\!\!\Big]_{R}) =$$
$$= \bigcup_{V^{C}(v)} ([\![r \in \underline{\mathrm{Dom}}_{R}(u)]\!]_{R} \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}v, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}) \cap$$
$$\cap [\![r = v]\!]_{R} \cap \Big[\!\!\Big[t = \underline{u(r)}_{R} \,\underline{\cap}_{R} \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z]\!]_{Q}, R, S) \Big]\!\!\Big]_{R}) =$$

$$= \bigcup_{V^C(v)} (\llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \llbracket r = v\rrbracket_R \cap \left\llbracket t = \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\right\rrbracket_R) =$$
$$= \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \bigcup_{V^C(v)} \llbracket r = v\rrbracket_R \cap \left\llbracket t = \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\right\rrbracket_R =$$
$$= \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \left\llbracket t = \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\right\rrbracket_R$$

$$V^C(r,t,u,S), V^D(Z) \vdash \llbracket \underline{\langle r, t\rangle}_R \in u_{\cap Z}\rrbracket_R =$$
$$= \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \left\llbracket t = \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\right\rrbracket_R \qquad (6)$$

$$V^C(r,u,S), V^D(Z) \vdash$$
$$\vdash \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \Big\| \underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R =$$
$$= \langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle \Big\|_R \subset'$$
$$\subset' \bigcup_{V^C(v)} (\llbracket v \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \Big\| \underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R =$$
$$= \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle \Big\|_R);$$
$$(3) \quad \underset{\underline{\langle r, \underline{u(r)}_R \underline{\cap}_R \mathrm{Sh}\mathring{(}\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R}{\overset{t}{\Longrightarrow}}$$
$$\Longrightarrow V^C(\underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R), V^C(u,S) \vdash$$
$$\vdash \left\llbracket \underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R \in u_{\cap Z}\right\rrbracket_R =$$
$$\subset' \bigcup_{V^C(v)} (\llbracket v \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap$$
$$\cap \Big\| \underline{\langle r, \underline{u(r)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle}_R =$$
$$= \langle v, \underline{u(v)}_R \,\underline{\cap}_R \mathrm{Sh}\mathring{(}\,\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q, R, S)\rangle \Big\|_R)$$

$$V^C(r,u,S), V^D(Z) \vdash$$
$$\vdash \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z\rrbracket_Q) \cap \qquad \Longrightarrow$$

$$\Longrightarrow \quad \cap \Big\| \underline{\langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R =$$

$$= \langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \Big\|_R \subset'$$

$$\subset' \Big[\!\Big[\underline{\langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R \in u_{\cap Z} \Big]\!\Big]_R$$

$$V^C(r,u,S) \vdash$$

$$\vdash |R| \subset' \Big\| \underline{\langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R =$$

$$= \langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle \Big\|_R;$$

$$V^C(r,u,S), V^D(Z) \vdash$$

$$\vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \subset'$$

$$\subset' \Big[\!\Big[\underline{\langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R \in u_{\cap Z} \Big]\!\Big]_R \qquad (7)$$

(6)

$$V^C(u,S), V^D(Z) \vdash |R| \subset' [\![\mathrm{Fn}(u_{\cap Z})]\!]_R; \qquad (7)$$

$$V^C(r,u,S), V^D(Z) \vdash$$

$$\vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \subset' [\![\mathrm{Fn}(u_{\cap Z})]\!]_R \cap$$

$$\cap \Big[\!\Big[\underline{\langle r, \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \rangle}_R \in u_{\cap Z} \Big]\!\Big]_R \subset'$$

$$\subset' \Big[\!\Big[\underline{u_{\cap Z}(r)}_R = \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \Big]\!\Big]_R$$

$$V^C(r,u,S), V^D(Z) \vdash$$

$$\vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \subset'$$

$$\subset' \Big[\!\Big[\underline{u_{\cap Z}(r)}_R = \underline{u(r)}_R \, \underline{\cap}_R \mathrm{Sh}\mathring{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \Big]\!\Big]_R \qquad (8)$$

(6)

$$V^C(r,u,S), V^D(Z,G) \vdash \overline{}_R\, \overline{}_R [\![r \in \underline{\mathrm{Dom}}_R(u_{\cap Z})]\!]_R =$$

$$= \overline{}_R\, \overline{}_R [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \overline{}_R\, \overline{}_R \mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q)$$

$$V^C(r,u,S), V^D(Z,G) \vdash \overline{}_Q\, \overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u_{\cap Z})]\!]_R =$$

$$= \overline{}_Q\, \overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \overline{}_Q\, \overline{}_Q \mathrm{h}(\mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q))$$

$$V^C(u,S), V^D(Z,G) \vdash \forall r (V^C(r) \to \overline{}_Q\, \overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u_{\cap Z})]\!]_R =$$

$$= \overline{}_Q\, \overline{}_Q [\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_R \cap \overline{}_Q\, \overline{}_Q \mathrm{h}(\mathrm{Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q))) \qquad (9)$$

$$(9)\Longrightarrow\quad V^{C}(u,S),V^{D}(Z)\vdash\forall r(V^{C}(r)\to\overline{\ }_{Q}\,\overline{\ }_{Q}[\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u_{\cap Z})]\!]_{R}=$$

$$=\overline{\ }_{Q}\,\overline{\ }_{Q}[\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)]\!]_{R}\cap\overline{\ }_{Q}\,\overline{\ }_{Q}\mathrm{h}(\mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))\in Z]\!]_{Q})))$$

$$V^{C}(u,S),V^{D}(t,Z,G)\vdash$$

$$\vdash\overline{\ }_{Q}\,\overline{\ }_{Q}[\![t\in\underline{\mathrm{In}}_{Q}(\mathrm{h}\ u_{\cap Z},\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}=$$

$$=\overline{\ }_{Q}\,\overline{\ }_{Q}\bigcup_{V^{C}(r)}(\overline{\ }_{Q}\,\overline{\ }_{Q}[\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u_{\cap Z})]\!]_{Q}\cap$$

$$\cap[\![\underline{\mathrm{Rg}}_{Q}(G)\cap\mathrm{h}\ \underline{u_{\cap Z}(r)}_{R}\neq 0]\!]_{Q}\cap$$

$$\cap[\![t=\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q})$$

$$V^{C}(u,S),V^{D}(t,Z)\vdash$$

$$\vdash\overline{\ }_{Q}\,\overline{\ }_{Q}[\![t\in\underline{\mathrm{In}}_{Q}(\mathrm{h}\ u_{\cap Z},\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}=$$

$$=\overline{\ }_{Q}\,\overline{\ }_{Q}\bigcup_{V^{C}(r)}(\overline{\ }_{Q}\,\overline{\ }_{Q}[\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)]\!]_{Q}\cap$$

$$\cap\overline{\ }_{Q}\,\overline{\ }_{Q}\mathrm{h}(\mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))\in Z]\!]_{Q}))\cap$$

$$\cap[\![\underline{\mathrm{Rg}}_{Q}(G)\cap\mathrm{h}\ \underline{u_{\cap Z}(r)}_{R}\neq 0]\!]_{Q}\cap$$

$$\cap[\![t=\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q})=$$

$$=\overline{\ }_{Q}\,\overline{\ }_{Q}\bigcup_{V^{C}(r)}([\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)]\!]_{Q}\cap$$

$$\cap\mathrm{h}(\mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))\in Z]\!]_{Q}))\cap$$

$$\cap[\![\underline{\mathrm{Rg}}_{Q}(G)\cap\mathrm{h}\ \underline{u_{\cap Z}(r)}_{R}\neq 0]\!]_{Q}\cap$$

$$\cap[\![t=\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q})$$

$$V^{C}(u,S),V^{D}(t,Z)\vdash$$

$$\vdash\overline{\ }_{Q}\,\overline{\ }_{Q}[\![t\in\underline{\mathrm{In}}_{Q}(\mathrm{h}\ u_{\cap Z},\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}=$$

$$=\overline{\ }_{Q}\,\overline{\ }_{Q}\bigcup_{V^{C}(r)}([\![\mathrm{h}r\in\mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)]\!]_{Q}\cap$$

$$\cap\mathrm{h}\ \mathrm{Dom}([\![\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))\in Z]\!]_{Q})\cap$$

$$\cap[\![\underline{\mathrm{Rg}}_{Q}(G)\cap\mathrm{h}\ \underline{u_{\cap Z}(r)}_{R}\neq 0]\!]_{Q}\cap$$

$$\cap[\![t=\underline{\mathrm{In}}_{Q}(\mathrm{h}r,\underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q})\qquad(10)$$

$$\frac{(8)}{\begin{aligned}&V^{C}(r,u,S), V^{D}(Z)\vdash\\ &\vdash \llbracket \mathrm{h}r \in \underline{\mathrm{Dom}}_{R}(u)\rrbracket_{R} \cap \mathrm{h}\ \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}) \subset'\\ &\subset' \left\llbracket \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} = \mathrm{h}\ \underline{u(r)}_{R}\ \underline{\cap}_{R} \mathrm{Sh}\mathring{(}\ \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}, R, S)\right\rrbracket_{R}\end{aligned}}$$

$$\begin{aligned}&V^{C}(u,S), V^{D}(Z,G)\vdash \forall r\,(V^{C}(r)\to\\ &\to \llbracket \mathrm{h}r \in \underline{\mathrm{Dom}}_{R}(u)\rrbracket_{R} \cap \mathrm{h}\ \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}) \subset'\\ &\subset' \left\llbracket \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} = \mathrm{h}\ \underline{u(r)}_{R}\ \underline{\cap}_{R} \mathrm{Sh}\mathring{(}\ \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}, R, S)\right\rrbracket_{R});\end{aligned}$$

$$\frac{\begin{aligned}&(10)\Longrightarrow\quad V^{C}(u,S), V^{D}(t,Z)\vdash\\ &\vdash \overline{\ }_{Q}\overline{\ }_{Q}\llbracket t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\ u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q} =\\ &= \overline{\ }_{Q}\overline{\ }_{Q}\bigcup_{V^{C}(r)}(\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)\rrbracket_{Q} \cap\\ &\qquad \cap \mathrm{h}\ \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}) \cap\\ &\qquad \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} \neq 0\rrbracket_{Q} \cap\\ &\qquad \cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q})\end{aligned}}{\begin{aligned}&V^{C}(u,S), V^{D}(t,Z)\vdash\\ &\vdash \overline{\ }_{Q}\overline{\ }_{Q}\llbracket t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\ u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q} =\\ &= \overline{\ }_{Q}\overline{\ }_{Q}\bigcup_{V^{C}(r)}(\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)\rrbracket_{Q} \cap\\ &\qquad \cap \mathrm{h}\ \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}) \cap\\ &\quad \cap \left\llbracket \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} = \mathrm{h}\ \underline{u(r)}_{R}\ \underline{\cap}_{R} \mathrm{Sh}\mathring{(}\ \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}, R, S)\right\rrbracket_{R} \cap\\ &\qquad \cap \llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} \neq 0\rrbracket_{Q} \cap\\ &\qquad \cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}) =\\ &= \overline{\ }_{Q}\overline{\ }_{Q}\bigcup_{V^{C}(r)}(\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_{R}(u)\rrbracket_{Q} \cap\\ &\qquad \cap \mathrm{h}\ \mathrm{Dom}(\llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}) \cap\\ &\quad \cap \left\llbracket \mathrm{h}\ \underline{u_{\cap Z}(r)}_{R} = \mathrm{h}\ \underline{u(r)}_{R}\ \underline{\cap}_{R} \mathrm{Sh}\mathring{(}\ \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}, R, S)\right\rrbracket_{R} \cap\\ &\quad \cap \left\llbracket \underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\ \underline{u(r)}_{R}\ \underline{\cap}_{R} \mathrm{Sh}\mathring{(}\ \llbracket \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G)) \in Z\rrbracket_{Q}, R, S) \neq 0\right\rrbracket_{Q} \cap\\ &\qquad \cap \llbracket t = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}) =\end{aligned}}$$

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$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, \mathrm{h\ Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \Big\|\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \cap_R \mathrm{Sh}\overset{\circ}{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \neq 0\Big\|_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$
$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, \mathrm{h\ Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \Big\|\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \cap_Q \mathrm{h\ Sh}\overset{\circ}{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \neq 0\Big\|_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$
$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, \mathrm{h\ Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap\, [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap \Big\|\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h\ Sh}\overset{\circ}{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \neq 0\Big\|_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$
$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, {}_{\overline{Q}}\,{}_{\overline{Q}}\mathrm{h\ Dom}([\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q) \cap$$
$$\cap \Big\|\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h\ Sh}\overset{\circ}{(}\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q, R, S) \neq 0\Big\|_Q \cap$$
$$\cap\, [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) = \quad {}^{2)}$$
$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, {}_{\overline{Q}}\,{}_{\overline{Q}}[\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q \cap$$
$$\cap\, [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$
$$= {}_{\overline{Q}}\,{}_{\overline{Q}}\bigcup_{V^C(r)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$
$$\cap\, [\![\underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G)) \in Z]\!]_Q \cap$$
$$\cap\, [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap\, [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}r, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) =$$

$$= \overline{_Q}\,\overline{_Q} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R}(u) \rrbracket_Q \cap$$

$$\cap \llbracket t \in Z \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}\, \underline{u(r)_R} \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}_Q}(\mathrm{h}r, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q) =$$

$$= \overline{_Q}\,\overline{_Q} (\llbracket t \in Z \rrbracket_Q \cap \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R}(u) \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}\, \underline{u(r)_R} \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}_Q}(\mathrm{h}r, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q)) =$$

$$= \overline{_Q}\,\overline{_Q} \llbracket t \in Z \rrbracket_Q \cap$$

$$\cap \overline{_Q}\,\overline{_Q} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}_R}(u) \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}_Q}(G) \cap \mathrm{h}\, \underline{u(r)_R} \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}_Q}(\mathrm{h}r, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q) =$$

$$= \overline{_Q}\,\overline{_Q} \llbracket t \in Z \rrbracket_Q \cap \overline{_Q}\,\overline{_Q} \llbracket t \in \underline{\mathrm{In}_Q}(\mathrm{h}u, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q =$$

$$= \overline{_Q}\,\overline{_Q} \llbracket t \in Z \underline{\cap_Q} \underline{\mathrm{In}_Q}(\mathrm{h}u, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q$$

$$V^C(u,S), V^D(t,Z) \vdash$$

$$\vdash \overline{_Q}\,\overline{_Q} \llbracket t \in \underline{\mathrm{In}_Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q = \overline{_Q}\,\overline{_Q} \llbracket t \in Z \underline{\cap_Q} \underline{\mathrm{In}_Q}(\mathrm{h}u, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q$$

$$V^C(u,S), V^D(Z) \vdash \forall t\, (V^D(t) \to$$

$$\to \overline{_Q}\,\overline{_Q} \llbracket t \in \underline{\mathrm{In}_Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q = \overline{_Q}\,\overline{_Q} \llbracket t \in Z \underline{\cap_Q} \underline{\mathrm{In}_Q}(\mathrm{h}u, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q)$$

$$V^C(u,S), V^D(Z) \vdash$$

$$\vdash |Q| \subset' \llbracket \underline{\mathrm{In}_Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}_Q}(G)) = Z \underline{\cap_Q} \underline{\mathrm{In}_Q}(\mathrm{h}u, \underline{\mathrm{Rg}_Q}(G)) \rrbracket_Q \qquad (11)$$

$$V^{C}(u,S), V^{D}(Z) \vdash |R| \subset' [\![\mathrm{Fn}(u_{\cap Z})\ \&\ u_{\cap Z} \subset' \underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S)]\!]_R$$

$$V^{C}(u,S), V^{D}(Z) \vdash [\![\mathrm{Fn}(u_{\cap Z})\ \&\ u_{\cap Z} \subset' \underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S)]\!]_R \subset'$$
$$\subset' \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R$$

$$V^{C}(u,S), V^{D}(Z) \vdash |R| \subset' \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R\,;$$

$$V^{C}(u,S) \vdash |R| \subset' [\![\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u)) = \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R$$

$$V^{C}(u,S), V^{D}(Z) \vdash |R| \subset' \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R \cap$$
$$\cap [\![\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u)) = \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R\,;$$

$$V^{C}(u,S), V^{D}(Z) \vdash \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R \cap$$
$$\cap [\![\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u)) = \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(u)\,\underline{\times}_R\,\underline{\mathrm{O}}_R(S))]\!]_R \subset'$$
$$\subset' \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_R$$

$$V^{C}(u,S), V^{D}(Z) \vdash |R| \subset' \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_R$$

$$V^{C}(u,S), V^{D}(Z) \vdash \mathrm{h}\,|R| \subset' \mathrm{h}\ \underset{R}{-}\underset{R}{-}[\![u_{\cap Z} \in \underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_R\,;$$

$$V^{C}(u,S), V^{D}(Z) \vdash \mathrm{h}\ \underset{R}{-}\underset{R}{-}[\![\mathrm{h}\,u_{\cap Z} \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_R =$$
$$= \underset{Q}{-}\underset{Q}{-}[\![\mathrm{h}\,u_{\cap Z} \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_Q$$

$$V^{C}(u,S), V^{D}(Z) \vdash \mathrm{h}\,|R| \subset' \underset{Q}{-}\underset{Q}{-}[\![\mathrm{h}\,u_{\cap Z} \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_Q$$

$$V^{C}(u,S), V^{D}(Z) \vdash |Q| \subset' \underset{Q}{-}\underset{Q}{-}[\![\mathrm{h}\,u_{\cap Z} \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\mathrm{P}}_{SR}(u))]\!]_Q \qquad (12)$$

$$V^{C}(u,S), V^{D}(Z) \vdash$$
$$\vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\,u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G)) = Z\,\underline{\cap}_Q\,\underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q$$

$$V^{C}(u,S), V^{D}(Z) \vdash$$
$$\vdash |Q| \subset' [\![\underline{\mathrm{In}}_Q(\mathrm{h}\,u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q$$

$$V^{C}(u,v,S), V^{D}(Z) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\,u_{\cap Z}) \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{[\![u_{\cap Z} \subset' u]\!]}_{SR} \neq 0]\!]_Q =$$
$$= V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\,u_{\cap Z}) \cap [\![\underline{\mathrm{In}}_Q(\mathrm{h}\,u_{\cap Z}, \underline{\mathrm{Rg}}_Q(G)) \subset' \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \quad ^{1)}$$

$$V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u, \mathrm{h}\,u_{\cap Z}) \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{[\![u_{\cap Z} \subset' u]\!]}_{SR} \neq 0]\!]_Q$$

$$V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' V_{R,S,Q}(\mathrm{h}\,u_{\cap Z})\,;$$

$$V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{[\![u_{\cap Z} \subset' u]\!]}_{SR} \neq 0]\!]_Q \qquad (13)$$

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$$V^{C}(u,S), V^{D}(Z) \vdash [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset'$$
$$\subset' [\![Z \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} ;$$
$$V^{C}(u,S), V^{D}(Z) \vdash [\![Z \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \subset'$$
$$\subset' [\![Z = Z \underline{\cap}_{Q} \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}$$

$$V^{C}(u,S), V^{D}(Z) \vdash [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset'$$
$$\subset' [\![Z = Z \underline{\cap}_{Q} \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} ;$$
$$V^{C}(u,S), V^{D}(Z) \vdash$$
$$\vdash |Q| \subset' [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G)) = Z \underline{\cap}_{Q} \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \quad {}^{2)}$$

$$V^{C}(u,S), V^{D}(Z) \vdash [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \cap |Q| \subset'$$
$$\subset' [\![Z = Z \underline{\cap}_{Q} \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \cap$$
$$\cap [\![\underline{\mathrm{In}}_{Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G)) = Z \underline{\cap}_{Q} \underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \subset'$$
$$\subset' [\![Z = \underline{\mathrm{In}}_{Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}$$

$$V^{C}(u,S), V^{D}(Z) \vdash [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset'$$
$$\subset' [\![Z = \underline{\mathrm{In}}_{Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \qquad (14)$$

$$(12);\ (13);\ (14)$$

$$V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset'$$
$$\subset' \overline{}_{Q}\,\overline{}_{Q} [\![\mathrm{h}\, u_{\cap Z} \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![u_{\cap Z} \subset' u]\!]_{SR}} \neq 0]\!]_{Q} \cap$$
$$\cap [\![Z = \underline{\mathrm{In}}_{Q}(\mathrm{h}\, u_{\cap Z}, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \subset'$$
$$\subset' \overline{}_{Q}\,\overline{}_{Q} \bigcup_{V^{C}(r)} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_{R}(\underline{\underline{\mathrm{P}}}_{SR}(u))]\!]_{Q} \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_{Q}(G) \cap \mathrm{h}\, \underline{[\![r \subset' u]\!]_{SR}} \neq 0]\!]_{Q} \cap$$
$$\cap [\![Z = \underline{\mathrm{In}}_{Q}(\mathrm{h}r, \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) = \quad {}^{3)}$$
$$= \overline{}_{Q}\,\overline{}_{Q} [\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}$$

$$V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset'$$
$$\subset' \overline{}_{Q}\,\overline{}_{Q} [\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \qquad (15)$$

$$
\begin{array}{l}
(15) \Longrightarrow V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \subset' \\
\qquad \subset' \underset{\overline{Q}}{}\,\underset{\overline{Q}}{}[\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \\
\hline
V^{C}(u,S), V^{D}(Z) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \cap \\
\qquad \cap \underset{\overline{Q}}{}[\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \subset' 0 \\
\hline
V^{C}(u,S) \vdash \forall Z (V^{D}(Z) \to \\
\qquad V_{R,S,Q}(\mathrm{h}u) \cap [\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \cap \\
\qquad \cap \underset{\overline{Q}}{}[\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \subset' 0) \\
\hline
V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \cap \\
\qquad \cap \bigcup_{V^{D}(Z)} ([\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \cap \\
\qquad \cap \underset{\overline{Q}}{}[\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) \subset' 0 \\
\hline
V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' \\
\qquad \subset' \underset{\overline{Q}}{}\bigcup_{V^{D}(Z)} ([\![Z \in \underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G)))]\!]_{Q} \cap \\
\qquad \cap \underset{\overline{Q}}{}[\![Z \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q}) \\
\hline
V^{C}(u,S) \vdash V_{R,S,Q}(\mathrm{h}u) \subset' \\
\subset' [\![\underline{\mathrm{P}}_{Q}(\underline{\mathrm{In}}_{Q}(\mathrm{h}u, \underline{\mathrm{Rg}}_{Q}(G))) \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(u), \underline{\mathrm{Rg}}_{Q}(G))]\!]_{Q} \qquad (16)
\end{array}
$$

Схема 2.6.22

$$
\begin{aligned}
&V^{C}(u), V^{D}(t) \vdash [\![t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \subset' \\
&\subset' \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{D}(r)} \big([\![r \in \underline{\mathrm{In}}_Q(\mathrm{h}u, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap [\![t \subset' \underline{\mathrm{V}_{r}}{}_Q]\!]_Q \big) \subset' \\
&\subset' \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{D}(r)} \Big(\overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![r = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \big) \cap \\
&\qquad\qquad\qquad \cap [\![t \subset' \underline{\mathrm{V}_{r}}{}_Q]\!]_Q \Big) = \\
&= \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{D}(r)} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![r = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![t \subset' \underline{\mathrm{V}_{r}}{}_Q]\!]_Q \big) = \\
&= \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{D}(r)} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![r = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \big) = \\
&= \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad\qquad\qquad \cap \bigcup_{V^{D}(r)} [\![r = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \big) = \\
&= \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad\qquad\qquad \cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \big)
\end{aligned}
$$

$$
\begin{aligned}
&V^{C}(u), V^{D}(t) \vdash [\![t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \subset' \\
&= \overline{{}_Q}\,\overline{{}_Q} \bigcup_{V^{C}(v)} \big([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap \\
&\qquad \cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap \\
&\qquad \cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}{}_Q]\!]_Q \big) \qquad (1)
\end{aligned}
$$

Схема 2.6.23

$$\frac{(1)}{}$$

$$V^{C}(u), V^{D}(t) \vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q \subset'$$

$$= V_{R,S,Q}(\mathrm{h}u) \cap \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q \cap$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}u) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}v) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}v) \cap$$

$$\cap \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} [\![\mathrm{h}v = \mathrm{h}r]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}v) \cap$$

$$\cap [\![\mathrm{h}v = \mathrm{h}r]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(v)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{V^{C}(v)} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap [\![\mathrm{h}v = \mathrm{h}r]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

$$\subset' \overline{Q}\,\overline{Q} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} ([\![\mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap \bigcup_{V^{C}(v)} [\![\mathrm{h}v = \mathrm{h}r]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0]\!]_Q$$

$$\cap [\![t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\, \underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q) \subset'$$

С х е м а 2.6.24

$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \rrbracket_Q)$$

$$V^C(u), V^D(t) \vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \rrbracket_Q \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \rrbracket_Q) \qquad (2)$$

$$(2)$$

$$V^C(u), V^D(t),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_v}}_{SR}, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \rrbracket_Q \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \subset' \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \rrbracket_Q \cap$$
$$\cap \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}r,\ \underline{\mathrm{Rg}}_Q(G))_Q}} \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_r}}_{SR}, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_r}}_{SR}, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{r \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_r}}_{SR}, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0 \rrbracket_Q$$
$$\cap \llbracket t \in \underline{\mathrm{P}}_Q(\underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_r}}_{SR}, \underline{\mathrm{Rg}}_Q(G))) \rrbracket_Q) \subset' \ {}^{1)}$$

$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t \in \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR}), \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \subset'$$

$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})) \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})(v)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q)) \subset'$$

$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})) \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})(v)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q)$$

$$V^C(u), V^D(t),$$

$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v) \subset'$$

$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\, \underline{\underline{\mathrm{V}}}_{v\;SR}, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}_Q \rrbracket_Q \subset'$$

$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}r \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(u) \rrbracket_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{u(r)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket \mathrm{h}v \in \mathrm{h}\, \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})) \rrbracket_Q \cap$$

$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\, \underline{\underline{\mathrm{P}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\;SR})(v)}_R \neq 0 \rrbracket_Q \cap$$

$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G)) \rrbracket_Q) \qquad (3)$$

Схема 2.6.26

$$V^C(r,v,S)\vdash$$
$$\vdash [\![v\in \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_R \subset' [\![\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})(v)_R = [\![v\subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}]\!]_R$$

$$V^C(r,v,S)\vdash$$
$$\vdash [\![\mathrm{h}v\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_R \subset' [\![\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})(v)_R = \mathrm{h}\ [\![v\subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}]\!]_Q\ ;(3)$$

$$V^C(u),V^D(t),V^C(S)$$
$$\forall v\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v)\subset'$$
$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\,\underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}}}_{v\,SR},\ \underline{\mathrm{Rg}}_Q(G))]\!]_Q)\vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u)\cap[\![t\in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\,\underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q \subset'$$
$$\subset' \overline{\ }_Q\overline{\ }_Q\bigcup_{V^C(r)}\bigcup_{V^C(v)}([\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap V_{R,S,Q}(\mathrm{h}r)\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ \underline{u(r)}_R\neq 0]\!]_Q\cap$$
$$\cap[\![\mathrm{h}v\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_Q\cap$$
$$\cap[\![\mathrm{h}\ \underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})(v)_R = \mathrm{h}\ [\![v\subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}]\!]_Q\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ \underline{\mathrm{P}}_R(\underline{\underline{\mathrm{V}}}_{r\,SR})(v)_R\neq 0]\!]_Q\cap$$
$$\cap[\![t=\underline{\mathrm{In}}_Q(\mathrm{h}v,\ \underline{\mathrm{Rg}}_Q(G))]\!]_Q)\subset'$$
$$\subset' \overline{\ }_Q\overline{\ }_Q\bigcup_{V^C(r)}\bigcup_{V^C(v)}([\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap V_{R,S,Q}(\mathrm{h}r)\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ \underline{u(r)}_R\neq 0]\!]_Q\cap$$
$$\cap[\![\mathrm{h}v\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_Q\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ [\![v\subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}\neq 0]\!]_Q\cap$$
$$\cap[\![t=\underline{\mathrm{In}}_Q(\mathrm{h}v,\ \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$$

$$V^C(u),V^D(t),V^C(S)$$
$$\forall v\in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v)\subset'$$
$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\,\underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}}}_{v\,SR},\ \underline{\mathrm{Rg}}_Q(G))]\!]_Q)\vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u)\cap[\![t\in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\,\underline{\mathrm{Rg}}_Q(G))}}_Q]\!]_Q \subset'$$
$$\subset' \overline{\ }_Q\overline{\ }_Q\bigcup_{V^C(r)}\bigcup_{V^C(v)}([\![\mathrm{h}r\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q\cap V_{R,S,Q}(\mathrm{h}r)\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ \underline{u(r)}_R\neq 0]\!]_Q\cap$$
$$\cap[\![\mathrm{h}v\in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_Q\cap$$
$$\cap[\![\underline{\mathrm{Rg}}_Q(G)\cap \mathrm{h}\ [\![v\subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}\neq 0]\!]_Q\cap$$
$$\cap[\![t=\underline{\mathrm{In}}_Q(\mathrm{h}v,\ \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \qquad (4)$$

С х е м а 2.6.27

$$V^C(r), V^C(u), V^C(S) \vdash$$

$$\vdash |R| \subset' [\![\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) = \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{r\,SR}) \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R$$ [1)];

$$V^C(r), V^C(u), V^C(S) \vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R \subset'$$

$$\subset' [\![\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{r\,SR}) \underline{\times}_R \underline{\mathrm{O}}_R(S)) \subset' \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_R$$

$$V^C(r), V^C(u), V^C(S) \vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R \subset'$$

$$\subset' [\![\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) = \underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{r\,SR}) \underline{\times}_R \underline{\mathrm{O}}_R(S))]\!]_R \cap$$

$$\cap [\![\underline{\mathrm{PF}}_R(\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{r\,SR}) \underline{\times}_R \underline{\mathrm{O}}_R(S)) \subset' \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_R \subset'$$

$$\subset' [\![\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) \subset' \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_R$$

$$V^C(r), V^C(u), V^C(S) \vdash [\![r \in \underline{\mathrm{Dom}}_R(u)]\!]_R \cap [\![V^{\underline{\mathrm{O}}_R(S)}(u)]\!]_R \subset'$$

$$\subset' [\![\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) \subset' \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_R$$

$$V^C(r), V^C(u), V^C(S) \vdash [\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}u) \subset'$$

$$\subset' [\![\mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) \subset' \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_Q;$$

(4)

$$V^C(u), V^D(t), V^C(S)$$

$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v) \subset'$$

$$\subset' [\![\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))_Q}} \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\,\underline{\underline{\mathrm{V}}}_{v\,SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \vdash$$

$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))_Q}}]\!]_Q \subset'$$

$$\subset' \overline{{}_Q}\,\overline{{}_Q}\bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{u(r)}_R \neq 0]\!]_Q \cap$$

$$\cap [\![\mathrm{h}v \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR}))]\!]_Q \cap$$

$$\cap [\![\mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{P}}}_{SR}(\underline{\underline{\mathrm{V}}}_{r\,SR})) \subset' \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{[\![v \subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \subset'$$

$$\subset' \overline{{}_Q}\,\overline{{}_Q}\bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{u(r)}_R \neq 0]\!]_Q \cap$$

$$\cap [\![\mathrm{h}v \in \mathrm{h}\,\underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}}}_{u\,SR})]\!]_Q \cap$$

$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\,\underline{[\![v \subset' \underline{\underline{\mathrm{V}}}_{r\,SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$

$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \subset'$$

$$\subset' \underline{\ }_{Q} \underline{\ }_{Q} \bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{SR})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \neq 0]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \subset'$$
$$\subset' \underline{\ }_{Q} \underline{\ }_{Q} \bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{SR})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \cap \mathrm{h}\ \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \subset'$$
$$\subset' \underline{\ }_{Q} \underline{\ }_{Q} \bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{SR})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \underline{\cap}_Q \mathrm{h}\ \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \subset'$$
$$\subset' \underline{\ }_{Q} \underline{\ }_{Q} \bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{SR})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \underline{\cap}_R \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q)$$

$$V^C(u), V^D(t), V^C(S)$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' [\![\underline{\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\, \underline{\mathrm{Rg}}_Q(G))}}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_v}}_{SR}, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap [\![t \in \underline{\underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\, \underline{\mathrm{Rg}}_Q(G))}}}_Q]\!]_Q \subset'$$
$$\subset' \underline{\ }_{Q} \underline{\ }_{Q} \bigcup_{V^C(r)} \bigcup_{V^C(v)} ([\![\mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)]\!]_Q \cap V_{R,S,Q}(\mathrm{h}r) \cap$$
$$\cap [\![\mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{SR})]\!]_Q \cap$$
$$\cap [\![\underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \underline{\cap}_R \underline{[\![v \subset' \underline{\underline{\mathrm{V}_r}}_{SR}]\!]_{SR}} \neq 0]\!]_Q \cap$$
$$\cap [\![t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))]\!]_Q) \qquad (5)$$

Схема 2.6.29

$$V^C(u), V^C(r), V^C(v), V^C(S) \vdash$$
$$\vdash \llbracket V^{\underline{\mathrm{O}}_R(S)}(u)\rrbracket_R \cap \llbracket r \in \underline{\mathrm{Dom}}_R(u)\rrbracket_R \cap \llbracket v \in \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})\rrbracket_R \subset'$$
$$\subset' \llbracket \underline{u(r)}_R \,\underline{\cap}_R \llbracket \underline{v \subset' \underline{\underline{\mathrm{V}_r}}_{\,SR}}\rrbracket_{SR} \subset' \underline{\underline{\underline{\mathrm{V}_u}}_{\,SR}(v)}_R\rrbracket_R$$

$$V^C(u), V^C(r), V^C(v), V^C(S) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)\rrbracket_Q \cap \llbracket \mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})\rrbracket_Q \subset'$$
$$\subset' \llbracket \mathrm{h}\ \underline{u(r)}_R \,\underline{\cap}_R \llbracket \underline{v \subset' \underline{\underline{\mathrm{V}_r}}_{\,SR}}\rrbracket_{SR} \subset' \mathrm{h}\ \underline{\underline{\underline{\mathrm{V}_u}}_{\,SR}(v)}_R\rrbracket_Q\ ;$$

(5)

$$V^C(u), V^D(t), V^C(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(\,V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\,\underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_v}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q\,) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\,\underline{\mathrm{Rg}}_Q(G))}}_Q\rrbracket_Q \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)\rrbracket_Q \cap$$
$$\cap \llbracket \mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})\rrbracket_Q \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{u(r)}_R \,\underline{\cap}_R \llbracket \underline{v \subset' \underline{\underline{\mathrm{V}_r}}_{\,SR}}\rrbracket_{SR} \neq 0\rrbracket_Q \cap$$
$$\cap \llbracket \mathrm{h}\ \underline{u(r)}_R \,\underline{\cap}_R \llbracket \underline{v \subset' \underline{\underline{\mathrm{V}_r}}_{\,SR}}\rrbracket_{SR} \subset' \mathrm{h}\ \underline{\underline{\underline{\mathrm{V}_u}}_{\,SR}(v)}_R\rrbracket_Q \cap$$
$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q) \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(r)} \bigcup_{V^C(v)} (\llbracket \mathrm{h}r \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(u)\rrbracket_Q \cap$$
$$\cap \llbracket \mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})\rrbracket_Q \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{\underline{\underline{\mathrm{V}_u}}_{\,SR}(v)}_R \neq 0\rrbracket_Q \cap$$
$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q) \subset'$$
$$\subset'_{\overline{Q}\,\overline{Q}} \bigcup_{V^C(v)} (\llbracket \mathrm{h}v \in \mathrm{h}\ \underline{\mathrm{Dom}}_R(\underline{\underline{\mathrm{V}_u}}_{\,SR})\rrbracket_Q \cap$$
$$\cap \llbracket \underline{\mathrm{Rg}}_Q(G) \cap \mathrm{h}\ \underline{\underline{\underline{\mathrm{V}_u}}_{\,SR}(v)}_R \neq 0\rrbracket_Q \cap$$
$$\cap \llbracket t = \underline{\mathrm{In}}_Q(\mathrm{h}v, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q) =$$
$$=_{\overline{Q}\,\overline{Q}} \llbracket t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q$$

$$V^C(u), V^D(t), V^C(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_R(u))(\,V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}v,\,\underline{\mathrm{Rg}}_Q(G))}}_Q \subset' \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_v}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q\,) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \cap \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_Q(\mathrm{h}u,\,\underline{\mathrm{Rg}}_Q(G))}}_Q\rrbracket_Q \subset'_{\overline{Q}\,\overline{Q}} \llbracket t \in \underline{\mathrm{In}}_Q(\mathrm{h}\ \underline{\underline{\mathrm{V}_u}}_{\,SR}, \underline{\mathrm{Rg}}_Q(G))\rrbracket_Q \qquad (6)$$

(6)

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$$V^{C}(u), V^{D}(t), V^{C}(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(u))(\, V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}v,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q} \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{v}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset' (\, \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q}\rrbracket_{Q} \supset_{Q} \overline{}_{Q}\, \overline{}_{Q} \llbracket t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{u}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,)$$

***

$$V^{C}(u), V^{C}(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(u))(\, V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}v,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q} \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{v}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,) \vdash$$
$$\vdash \forall t\, (\, V^{D}(t) \rightarrow$$
$$\rightarrow V_{R,S,Q}(\mathrm{h}u) \subset' (\, \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q}\rrbracket_{Q} \supset_{Q} \overline{}_{Q}\, \overline{}_{Q} \llbracket t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{u}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,))$$

***

$$V^{C}(u), V^{C}(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(u))(\, V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}v,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q} \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{v}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$
$$\subset' \bigcap_{V^{D}(t)}^{R} (\, \llbracket t \in \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q}\rrbracket_{Q} \supset_{Q} \overline{}_{Q}\, \overline{}_{Q} \llbracket t \in \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{u}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,)$$

***

$$V^{C}(u), V^{C}(S),$$
$$\forall v \in \mathrm{Dom}(\underline{\mathrm{Dom}}_{R}(u))(\, V_{R,S,Q}(\mathrm{h}v) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}v,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q} \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{v}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q}\,) \vdash$$
$$\vdash V_{R,S,Q}(\mathrm{h}u) \subset'$$
$$\subset' \llbracket \underline{\mathrm{V}_{\underline{\mathrm{In}}_{Q}(\mathrm{h}u,\, \underline{\mathrm{Rg}}_{Q}(G))}}_{Q} \subset' \underline{\mathrm{In}}_{Q}(\mathrm{h}\, \underline{\underline{\mathrm{V}_{u}}}_{\,S\,R}, \underline{\mathrm{Rg}}_{Q}(G))\rrbracket_{Q} \qquad (7)$$

Заключение

В отличие от классической математики, в интуиционизме невозможно провозгласить существование множества аксиоматически. Существование множества в интуиционизме означает возможность указания точного способа эффективного построения этого множества ; этот принцип в полной мере реализован в интуиционистской теории множеств **K**. Поэтому, большинство утверждений, рассмотренных в классическом методе форсинга, являются абсолютно неприемлемыми с точки зрения интуиционизма. К числу таких утверждений относится и континуум – гипотеза. У нас нет никаких оснований полагать, что множество действительных чисел равномощно некоторому ординалу, и уж тем более первому несчетному кардиналу. Единственной, приемлемой для интуиционизма, формой континуум – гипотезы является следующее утверждение:

$$\vdash \forall u \in \mathrm{P}({}^{\omega}\omega)\,(\neg\,\omega \Rightarrow u \to \neg\neg\, u \approx {}^{\omega}2),$$

являющееся естественным и осмысленным с точки зрения математической реальности, поскольку оно выполняется для достаточно широкого класса множеств, а именно аналитических множеств. Так, в интуиционистской теории множеств **K** доказуема следующая секвенция:

$$\mathrm{AC}_{\omega} \vdash \forall u \in \mathbf{\Sigma}^1_1 \cap \mathrm{P}({}^{\omega}\omega)\,(\neg\,\omega \Rightarrow u \to \neg\neg\, u \approx {}^{\omega}2),$$

являющаяся следствием теоремы Суслина о совершенном ядре аналитических множеств, доказательство которой в интуиционистской теории множеств **K** автор намерен привести в четвертом томе

$$({}^{\mathrm{Df}}\vdash \mathrm{AC}_u \sim$$

$$\sim \forall X \subset' u \times \mathrm{P}(u)(\mathrm{Dom}(X) = u \to \neg\neg\exists \mathrm{f} \in \mathrm{P}(X)(\mathrm{f} : u \to \mathrm{P}(u)))\ ).$$

Таким образом, слабая форма континуум – гипотезы

$$\vdash \forall u \in \mathrm{P}({}^{\omega}\omega)\,(\neg\, u \Rightarrow \omega \to \neg\neg\, u \approx {}^{\omega}2)$$

является кандидатом номер один на рассмотрение в интуиционистском методе форсинга.

Следующий этап применения интуиционистского метода форсинга – дескриптивная теория множеств. Следует отметить, что все теоремы классической дескриптивной теории множеств имеют аналоги в интуиционистской теории множеств **K**. Доказательства основных теорем дескриптивной теории множеств в интуиционистской теории множеств **K** автор намерен привести в четвертом томе.

Задачи, стоявшие перед классической дескриптивной теорией множеств, которые впоследствии были успешно решены, сохраняются в интуиционистской теории множеств **K** и ждут своего решения. Большая часть этих задач была поставлена Н.Н.Лузиным в ходе развития классической дескриптивной теории множеств. Эти задачи исходили из самой сущности дескриптивной теории множеств, и были такими же естественными, как и сама дескриптивная теория множеств.

Представляет большой интерес трансформация в интуиционистскую теорию множеств **K** исследований В.Г.Кановея в области классической дескриптивной теории множеств. Однако, основная трудность на этом пути – невозможность использования моделей интуиционистской теории множеств **K**, о которой говорилось ранее.

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Оглавление